Math 173 - Quiz 5

February 26, 2015

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Find the curvature function for the curve described by

$$\vec{r}(t) = 2t\hat{\imath} + t^2\hat{\jmath} - \frac{1}{3}t^3\hat{k}.$$

$\vec{r}\,'(t) = 2\hat{\imath} + 2t\hat{\jmath} - t^2\hat{k}$

$\vec{r}\,''(t) = 2\hat{\jmath} - 2t\hat{k}$

$$\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 2t & -t^2 \\ 0 & 2 & -2t \end{vmatrix}$$

$= -2t^2\hat{\imath} + 4t\hat{\jmath} + 4\hat{k}$

$\|\vec{r}\,' \times \vec{r}\,''\| = \sqrt{4t^4 + 16t^2 + 16} = 2(t^2+2)$

$\|\vec{r}\,'\| = \sqrt{t^4 + 4t^2 + 4} = t^2 + 2$

$$\boxed{K(t) = \frac{2(t^2+2)}{(t^2+2)^3} = \frac{2}{(t^2+2)^2}}$$

2. (4 points) Find the arc length function $s(t)$ for the line segment described by

$$\vec{r}(t) = t\hat{\imath} + (t+2)\hat{\jmath} + (3t-5)\hat{k}, \quad 0 \le t \le 1$$

and write $\vec{r}$ in terms of the parameter s.

$\vec{r}\,'(t) = \hat{\imath} + \hat{\jmath} + 3\hat{k}$ $\quad \|\vec{r}\,'\| = \sqrt{11}$

$$s(t) = \int_0^t \|\vec{r}\,'(\tau)\|\, d\tau$$

$$= \int_0^t \sqrt{11}\, d\tau$$

$= \sqrt{11}\, t$

$\boxed{s(t) = \sqrt{11}\, t} \Rightarrow t = \frac{s}{\sqrt{11}}$

$$\boxed{\vec{r}(s) = \frac{s}{\sqrt{11}}\hat{\imath} + \left(\frac{s}{\sqrt{11}} + 2\right)\hat{\jmath} + \left(\frac{3s}{\sqrt{11}} - 5\right)\hat{k} \quad 0 \le s \le \sqrt{11}}$$

3. (2 points) Determine the domain of $f(x,y) = \sqrt{x^3 + y}$.

$x^3 + y \ge 0 \Rightarrow y \ge -x^3$

$$\boxed{\text{Domain} = \{(x,y) : y \ge -x^3\}}$$