Math 173 - Quiz 11

Cinco de Mayo, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. Once you set up your integrals, you may use a CAS to evaluate them.

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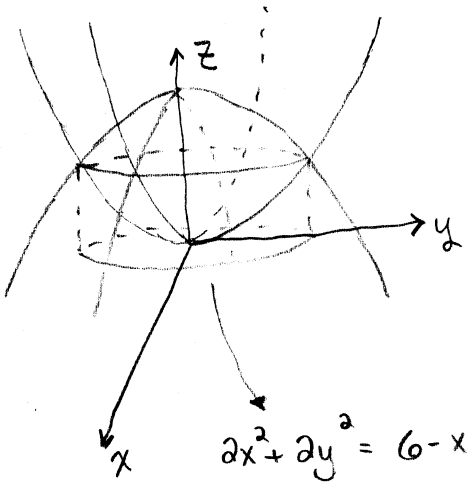
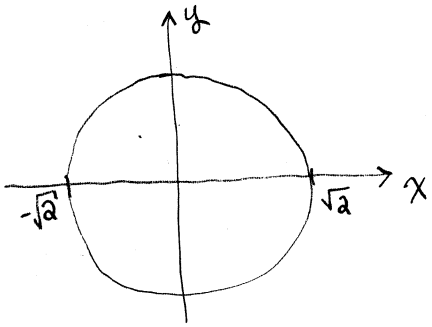
1. (5 points) Let E be the space region bounded by the paraboloids $z = 2x^2 + 2y^2$ and $z = 6 - x^2 - y^2$. Evaluate the following triple integral. (Hint: Use cylindrical coordinates.)

$$\iiint_E (x^2 + y^2)\, dV$$



$2x^2 + 2y^2 = 6 - x^2 - y^2$

$x^2 + y^2 = 2$



$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\sqrt{2}} \int_{z=2r^2}^{z=6-r^2} r^2 \, r \, dz \, dr \, d\theta$$

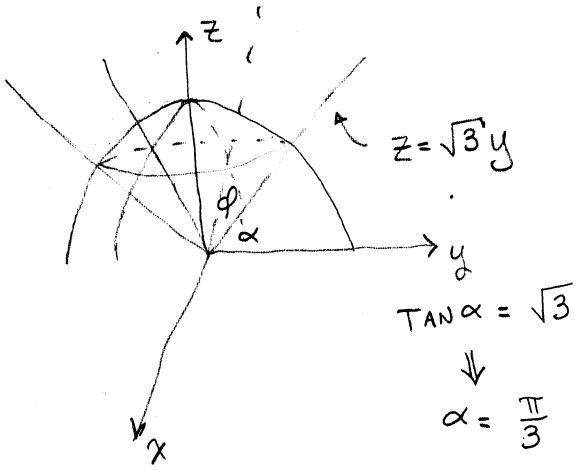
$$= 4\pi$$

Over ⟶

$z^2 = 3x^2 + 3y^2$

2. (5 points) Let U be the "ice cream cone" bounded below by $z = \sqrt{3(x^2+y^2)}$ and above by $x^2+y^2+z^2 = 4$. Find the vloume of U. (Hint: Use spherical coordinates.)

$\rho = 2$



$\tan\alpha = \sqrt{3}$

$\Downarrow$

$\alpha = \frac{\pi}{3}$

$$\iiint_U dV = \int_{\theta=0}^{2\pi} \int_{\rho=0}^{2} \int_{\varphi=0}^{\pi/6} \rho^2 \sin\varphi \, d\varphi \, d\rho \, d\theta$$

$$= \frac{8\pi}{3}(2-\sqrt{3})$$

$$\approx 2.245$$

$x^2+y^2+z^2 = 4$

$z^2 = 3x^2 + 3y^2$

$r = 1$

$\sin\varphi = \frac{1}{2}$

$\Rightarrow \varphi = \frac{\pi}{6}$