Math 173 - Quiz 2

Name key

January 28, 2016

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the angle between the vectors $\vec{u} = \hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ and $\vec{v} = 4\hat{\imath} - 4\hat{\jmath} + 3\hat{k}$.

$\vec{u} \cdot \vec{v} = (1)(4) + (-2)(-4) + (5)(3) = 27$

$\|\vec{u}\| = \sqrt{(1)^2 + (-2)^2 + (5)^2} = \sqrt{30}$

$\|\vec{v}\| = \sqrt{(4)^2 + (-4)^2 + (3)^2} = \sqrt{41}$

$\cos\theta = \dfrac{27}{\sqrt{30}\sqrt{41}} \Rightarrow \theta \approx 0.692176$ or $\theta \approx 39.66°$

2. (3 points) Let $\vec{u} = -2\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$. Find a unit vector that is orthogonal to $\vec{u}$.

$\vec{v} = v_1\hat{\imath} + v_2\hat{\jmath} + v_3\hat{k}$

$\vec{u} \cdot \vec{v} = -2v_1 + 5v_2 - 3v_3 = 0$

Make up some values for v_1, v_2, v_3

How about $v_1 = v_2 = v_3 = 1$.

$\vec{v} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\|\vec{v}\| = \sqrt{1+1+1} = \sqrt{3}$

$\dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{1}{\sqrt{3}}\hat{\imath} + \dfrac{1}{\sqrt{3}}\hat{\jmath} + \dfrac{1}{\sqrt{3}}\hat{k}$

3. (4 points) Let P and Q be the points $(3, 0, 2)$ and $(5, -2, -3)$, respectively. Find the projection of the vector $\vec{PQ}$ onto the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.

($\hat{\imath} + \hat{\jmath} + \hat{k} = \vec{v}$)

$\vec{PQ} = (5-3)\hat{\imath} + (-2-0)\hat{\jmath} + (-3-2)\hat{k}$

$= 2\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

$\text{proj}_{\vec{v}}\,\vec{PQ} = \dfrac{\vec{PQ} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\vec{v} = \dfrac{2-2-5}{1+1+1}\vec{v}$

$= \dfrac{-5}{3}\vec{v} = -\dfrac{5}{3}\hat{\imath} - \dfrac{5}{3}\hat{\jmath} - \dfrac{5}{3}\hat{k}$