

Math 173 - Quiz 3

February 4, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Find a unit vector orthogonal to both
- $\vec{u} = -3\hat{i} + 8\hat{j} + \hat{k}$
- and
- $\vec{w} = 5\hat{i} - 2\hat{j} - 2\hat{k}$
- .

$$\vec{u} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 8 & 1 \\ 5 & -2 & -2 \end{vmatrix} = \hat{i}(-16+2) - \hat{j}(-6-5) + \hat{k}(6-40) \\ = -14\hat{i} - \hat{j} - 34\hat{k}$$

$$\|\vec{u} \times \vec{w}\| = \sqrt{(-14)^2 + (-1)^2 + (-34)^2} = \sqrt{1353}$$

$$\frac{\vec{u} \times \vec{w}}{\|\vec{u} \times \vec{w}\|} =$$

$$= \frac{-1}{\sqrt{1353}} (14\hat{i} + \hat{j} + 34\hat{k})$$

2. (2 points) Find the area of the parallelogram determined by the vectors in Problem #1.

$$\text{Area} = \|\vec{u} \times \vec{w}\| = \sqrt{1353} \approx 36.78$$

3. (2 points) Find symmetric equations for the line through
- $P(1, 2, 3)$
- and
- $Q(-2, 5, 2)$
- .

$$\vec{v} = \vec{PQ} = -3\hat{i} + 3\hat{j} - \hat{k}$$

$$\text{Using } \vec{v} \text{ \& } (1, 2, 3)$$

$$\frac{x-1}{-3} = \frac{y-2}{3} = \frac{z-3}{-1}$$

4. (2 points) Find a unit vector parallel to the line with parametric equations:

$$x = 4 - t, \quad y = 4t, \quad z = 6 + 2t.$$

$$\vec{v} = -\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\|\vec{v}\| = \sqrt{(-1)^2 + (4)^2 + (2)^2} = \sqrt{21}$$

$$\frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{21}} (-\hat{i} + 4\hat{j} + 2\hat{k})$$