

Math 173 - Quiz 5
February 25, 2016

Name key Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) A projectile is falling in such a way that its motion is described by

$$\vec{r}(t) = (25\sqrt{3}t)\hat{i} + (-16t^2 + 25t + 25)\hat{j}.$$

Set up the definite integral that gives the length of the path of the projectile from $t = 0$ until it reaches its max height. Use your calculator or a computer algebra system to approximate the value of your integral.

$$\begin{aligned} -32t + 25 &= 0 \\ \Rightarrow t &= \frac{25}{32} \end{aligned}$$

$$\text{Arc Length} = \int_0^{25/32} \sqrt{(25\sqrt{3})^2 + (-32t + 25)^2} dt$$

$$\approx 35.624 \text{ FT}$$

2. (4 points) At what point does the graph of $y = x^2$ have the greatest curvature? Find a set of parametric equations for the graph. Then compute the curvature at the point of greatest curvature.

THE GREATEST
CURVATURE IS
PRETTY CLEARLY
AT (0,0).

$$\vec{r}(t) = t\hat{i} + t^2\hat{j}$$

$$\vec{r}'(t) = \hat{i} + 2t\hat{j}$$

$$\hat{T}(t) = \frac{\hat{i} + 2t\hat{j}}{\sqrt{1+4t^2}}$$

$$\hat{T}'(t) = \frac{(1+4t^2)^{1/2}(2\hat{j}) - (\hat{i} + 2t\hat{j})(\frac{1}{2})(1+4t^2)^{-1/2}(8t)}{1+4t^2}$$

$$\hat{T}'(0) = \frac{2\hat{j}}{1}$$

$$K = \frac{1}{\|\vec{r}'(0)\|} \|\hat{T}'(0)\| = 2$$

3. (3 points) Reparameterize the position vector below in terms of the arc length parameter.

$$\vec{r}(t) = (2+3t)\hat{i} - 5t\hat{j} + (1+t)\hat{k}, \quad 0 \leq t \leq 2$$

$$\begin{aligned} S &= \int_0^t \sqrt{3^2 + (-5)^2 + 1^2} d\tau \\ &= \sqrt{35}t \end{aligned}$$

$$t = \frac{s}{\sqrt{35}}$$

$$\vec{r}(s) = \left(2 + \frac{3s}{\sqrt{35}}\right)\hat{i} - \frac{5s}{\sqrt{35}}\hat{j} + \left(1 + \frac{s}{\sqrt{35}}\right)\hat{k},$$

$$0 \leq s \leq 2\sqrt{35}$$