Math 173 - Quiz 7

March 17, 2016

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the directional derivative of $f(x,y,z) = x^2yz - xy^2z + xyz^2$ at the point $(1,2,3)$ in the direction of $\vec{w} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

$$\vec{\nabla} f(x,y,z) = (2xyz - y^2z + yz^2)\hat{\imath} + (x^2z - 2xyz + xz^2)\hat{\jmath} + (x^2y - xy^2 + 2xyz)\hat{k}$$

$$\vec{\nabla} f(1,2,3) = (12 - 12 + 18)\hat{\imath} + (3 - 12 + 9)\hat{\jmath} + (2 - 4 + 12)\hat{k} = 18\hat{\imath} + 10\hat{k}$$

$$D_{\vec{v}} f(1,2,3) = \frac{1}{\|\vec{w}\|}\vec{\nabla} f(1,2,3)\cdot\vec{w} = \frac{74}{\sqrt{9+1+4}} = \boxed{\frac{74}{\sqrt{14}}}$$

2. (5 points) Use the definition of differentiable to show that $f(x,y) = 2xy + y^2$ is differentiable at any point (x,y).

$$f_x(x,y) = 2y, \quad f_y(x,y) = 2x + 2y$$

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y) = 2(x+\Delta x)(y+\Delta y) + (y+\Delta y)^2 - 2xy - y^2$$

$$= \cancel{2xy} + \underline{\underline{2x\Delta y}} + \underline{2y\Delta x} + \underline{\underline{\underline{2\Delta x\Delta y}}} + \cancel{y^2} + \underline{\underline{2y\Delta y}} + \underline{\underline{\underline{\Delta y^2}}} - \cancel{2xy} - \cancel{y^2}$$

$$= \underset{f_x}{2y}\,\Delta x + \underset{f_y}{(2x+2y)}\,\Delta y + \underset{\epsilon_1}{2\Delta y}\,\Delta x + \underset{\epsilon_2}{\Delta y}\,\Delta y$$

Δz HAS CORRECT FORM AND $\epsilon_1, \epsilon_2 \to 0$ AS $(\Delta x, \Delta y) \to (0,0)$.

3. (2 points) Assume that $\underbrace{x^3e^{xz} + \ln(xyz) = 8}$ implicitly defines z as a function of x and y. Find $\partial z/\partial x$.

$$F(x,y,z) = x^3e^{xz} + \ln(xyz) - 8$$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = \boxed{\frac{-\left(3x^2e^{xz} + x^3ze^{xz} + \frac{1}{x}\right)}{x^4e^{xz} + \frac{1}{z}}}$$