Math 173 - Quiz 8

April 7, 2016

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Find and classify the critical points of $f(x, y) = y^4 - 4xy + x^2 + 7$.

$f_x(x,y) = -4y + 2x = 0 \Rightarrow 4x = 8y$

$f_y(x,y) = 4y^3 - 4x = 0$

$4y^3 - 8y = 4y(y^2 - 2) = 0$

$y = 0, \quad y = \pm\sqrt{2}$

$x = 0 \qquad x = \pm 2\sqrt{2}$

$(0,0), \ (2\sqrt{2}, \sqrt{2}), \ (-2\sqrt{2}, -\sqrt{2})$

$$d(x,y) = \begin{vmatrix} 2 & -4 \\ -4 & 12y^2 \end{vmatrix} = 24y^2 - 16$$

$d(0,0) = -16 \Rightarrow (0, 0, f(0,0)) = (0,0,7)$ IS A SADDLE PT

$d(\pm 2\sqrt{2}, \pm\sqrt{2}) = 32$ & $f_{xx}(\pm 2\sqrt{2}, \pm\sqrt{2}) > 0$

$\Rightarrow f(\pm 2\sqrt{2}, \pm\sqrt{2}) = 3$ IS A REL MIN

2. (5 points) Use Lagrange multipliers to find the extreme values of $f(x, y) = xy$ subject to $3x^2 + y^2 = 6$.

$$\vec{\nabla} f(x,y) = y\hat{i} + x\hat{j}$$

$$\vec{\nabla} g(x,y) = 6x\hat{i} + 2y\hat{j}$$

$$\left.\begin{aligned} y &= \lambda 6x \\ x &= \lambda 2y \end{aligned}\right\} \quad 2\lambda y^2 = 6\lambda x^2 \Rightarrow \lambda = 0 \text{ OR } y^2 = 3x^2$$

$$3x^2 + y^2 = 6$$

$\lambda = 0 \Downarrow$ $x = 0$, $y = 0$ Impossible

$y^2 = 3x^2 \Downarrow$ $6x^2 = 6 \Downarrow$ $x^2 = 1$, $x = \pm 1$

$x = 1$, $y = \pm\sqrt{3}$

$x = -1$, $y = \pm\sqrt{3}$

$(1, \sqrt{3})$, $(1, -\sqrt{3})$, $(-1, \sqrt{3})$, $(-1, -\sqrt{3})$

$$\boxed{\begin{aligned} f(\pm 1, \pm\sqrt{3}) &= \sqrt{3} \quad \text{MAX} \\ f(\pm 1, \mp\sqrt{3}) &= -\sqrt{3} \quad \text{MIN} \end{aligned}}$$