

Math 173 - Test 1
February 18, 2016

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) Let $\vec{v} = \hat{i} + 2\hat{j} - 7\hat{k}$ and $\vec{w} = 3\hat{i} - \hat{j} - \hat{k}$. Find a unit vector in the direction of $3\vec{w} - \vec{v}$.

2. (2 points) What does it mean for two vectors to be orthogonal?

3. (5 points) Find a 2D unit vector that is normal (perpendicular) to the graph of $y = x^3$ at the point where $x = -2$.

4. (4 points) Consider the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = -6\hat{i} - 9\hat{j} + \hat{k}$. Carefully explain why these vectors are not parallel. Then make changes to \vec{b} so that \vec{a} and your new \vec{b} are parallel.

5. (6 points) Consider the line segment connecting the point $P(2, 1, 5)$ to the point $Q(8, -2, 3)$.

(a) Find the midpoint of the segment.

(b) Using the midpoint as your initial point, find a set of parametric equations for the line **segment**.

6. (5 points) Find an equation of the plane passing through the point $(1, 2, 3)$ and normal to the line with symmetric equations

$$\frac{x + 9}{2} = \frac{y - 3}{-5} = z.$$

7. (5 points) Let $\vec{u} = 3\hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{v} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Find the length of the projection of \vec{u} onto \vec{v} .

8. (5 points) Find $\vec{r}(t)$ that satisfies these conditions:

$$\vec{r}'(t) = \frac{1}{1+t^2}\hat{i} + \frac{1}{t^2}\hat{j} + \frac{1}{t}\hat{k}, \quad \vec{r}(1) = 2\hat{i}$$

9. (4 points) Sketch a diagram that shows two vectors, \vec{v} and \vec{w} , and then show the vector $\text{proj}_{\vec{v}}\vec{w}$.

10. (8 points) Consider the vectors $\vec{u} = -2\hat{i} + \hat{j} - 6\hat{k}$ and $\vec{v} = -3\hat{i} - 3\hat{j} + 8\hat{k}$.

(a) Find a vector that is orthogonal to both \vec{u} and \vec{v} .

(b) Find the area of the triangle determined by the vectors \vec{u} and \vec{v} .

11. (8 points) Find the measure of the angle between the planes described by the equations below.

$$3x - 2y + 10z = 0$$

$$5x + 7y + z = 10$$

12. (12 points) In a field goal attempt on a flat field, a football is kicked off the ground at an angle of 30° to the horizontal with an initial speed of 65 ft/sec.

(a) What horizontal distance does the football travel while it is in the air?

(b) To score a field goal, the ball must clear the cross bar of the goal post, which is 10 ft above the ground. What is the furthest from the goal post the kick can originate and score a field goal?

13. (4 points) Find a vector-valued function whose graph is the parabola given by $y = x^2 + 1$.

14. (12 points) An object is moving in such a way that its position at time t is given by

$$\vec{r}(t) = \sin 3t \hat{i} - \cos 3t \hat{j} + 2t^2 \hat{k}.$$

(a) Determine the function that gives the speed of the object at time t .

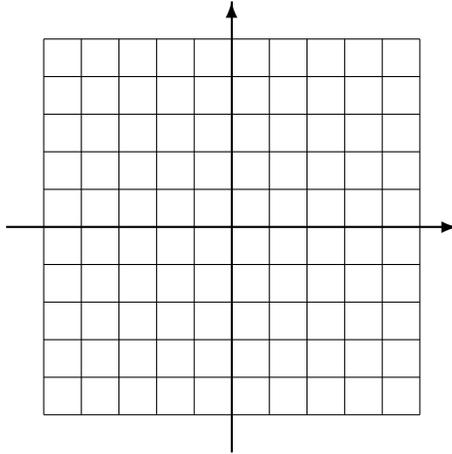
(b) Find the speed at time $t = 5$.

(c) Find $\hat{T}(t)$, the unit tangent vector for $\vec{r}(t)$.

(d) What would you find if you computed $\hat{T}(t) \cdot \hat{T}'(t)$? Explain.

15. (8 points) Consider the vector-valued function $\vec{r}(t) = 2t\hat{i} - t^2\hat{j}$.

(a) Sketch the graph of $\vec{r}(t)$.



(b) Without computing $\hat{T}(t)$, sketch the vector $\hat{T}(2)$ on your graph.

(c) How would your sketch of $\hat{T}(2)$ be different if the first component of $\vec{r}(t)$ was $-2t$ rather than $2t$? Explain.

16. (7 points) Let $\vec{r}(t) = -7\cos t\hat{i} - 7\sin t\hat{j} + t\hat{k}$. Compute $\hat{N}(t)$.