Math 173 - Test 2

March 31, 2016

Name _____

Score _____

Show all work. Supply explanations when necessary.
YOU MUST WORK INDIVIDUALLY ON THIS TEST.

1. (10 points) Determine the limit or show that it does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(2,0)} \frac{(x-y)^2+2(x-y)-x^2-2x}{y}$

   (b) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{xy^3-8x^2}{xy^2-4}$

2. (5 points) Consider the surface described by the equation $4x^2 - y^2 + 9z^2 = 4$.

   (a) Fix a value for one of the variables and draw a good sketch of the corresponding level curve.

   (b) Fix a value for one of the other variables and briefly describe the corresponding level curve.

   (c) Identify the surface.

3. (5 points) Draw a reasonably good sketch of the graph of $f(x, y) = 1 - x^2 - y^2$ and identify the surface.

4. (8 points) Use differentials to estimate the change in $f(x, y, z) = x^2 \ln(5yz + 1)$ as (x, y, z) changes from $(2, 1, 3)$ to $(1.99, 1.02, 3.05)$.

5. (8 points) Suppose the equation $xyz + e^{-xz^2} + 5\cos 2xy = 6$ implicitly defines z as a function of x and y. Find $\partial z/\partial y$ at the point $(0, 3, 1)$.

6. (10 points) Consider the function $g(x, y, z) = x \tan^{-1}(y/z)$.

   (a) Find the directional derivative of g at the point $(1, 2, -2)$ in the direction of $\vec{w} = \hat{\imath} + \hat{\jmath} - \hat{k}$.

   (b) At the point $(1, 2, -2)$, what is the direction of steepest descent (maximum decrease) of g?

7. (6 points) Let $g(x, y, z) = x \cos(y^2 z + yz^2) - x^2 e^{y^2 z} + x\sqrt{4y + 2z}$. Suppose you'd like to determine the mixed partial derivative g_{yzxx}. Which mixed partial derivative is equal to g_{yzxx}, but would be much simpler to compute? Compute that partial derivative.

8. (10 points) Find and classify the critical points of $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$.

9. (8 points) Consider the function $h(x, y) = \sqrt{1 + x - y^2}$.

(a) What is the domain of h?

(b) Discuss the continuity of h.

(c) Sketch the level curve $h(x, y) = 0$.

(d) The graph of h is one-half of one of the quadric surfaces that we are familiar with. Describe the graph of h.

10. (10 points) A surface is described by the equation $z + 1 = xe^y \cos z$. Find equations for the tangent plane and normal line through $(1, 0, 0)$.

(Intentionally blank.)

11. (10 points) At what point does the function $f(x) = e^x$ have the greatest curvature? (Hint: Compute the curvature function and find its maximum value.)

12. (10 points) Suppose we are given the two functions

$$f(x, y) = xy + x - 4y - 10, \qquad g(x, y) = xy - 6x + y - 8$$

and we wish to solve the system of nonlinear equations $f(x, y) = 0$, $g(x, y) = 0$. We can approximate a solution using linearizations and Newton's method.

(a) Let $(x_0, y_0) = (-1, -2)$ be our initial guess at the solution. Let $f_0(x, y)$ and $g_0(x, y)$ be the linearizations of f and g at the point (x_0, y_0). Find $f_0(x, y)$ and $g_0(x, y)$.

(b) Solve the linear system of equations

$$f_0(x, y) = 0, \qquad g_0(x, y) = 0.$$

(c) Let (x_1, y_1) be the solution of the linear system in part (b). It represents an improved guess at the solution of the original system. Compute $f(x_1, y_1)$ and $g(x_1, y_1)$.

(d) (Extra Credit 4 pts) On a separate sheet of paper, use (x_1, y_1) in place of (x_0, y_0) and repeat the steps above to further improve our guess at the solution.