Math 173 - Test 3a

April 28, 2016

Name ______________________

Score ____________

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due Tuesday, May 3. You must work individually on this test—you will not be given any credit for group work.

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1. (12 points) Let E be the plane region between the graphs of $y = x^2$ and $y = x + 2$. Sketch the region E, write the iterated integrals (both orders of integration) associated with the double integral

$$\iint_E (xy + 5)\, dA,$$

and evaluate either one of the iterated integrals. Carry out the integration by hand, but you may use a CAS to check your work.

2. (12 points) Use the method of Lagrange multipliers to find the extreme values of $f(x,y) = xy$ subject to the constraint

$$\frac{x^2}{8} + \frac{y^2}{2} = 1.$$

Follow-up: Sketch the graph of the constraint equation as well as several of the level curves of $f(x,y)$ (i.e., $xy = \pm k$). Mark your critical points on the graph. (If you sketch these graphs by hand, do so carefully! You may use technology such as GeoGebra if you prefer.)

3. (8 points) Sketch the region of integration. Then write the iterated integral with the reversed order of integration. You need not evaluate the integral.

$$\int_0^{\sqrt{2}} \int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} y\, dx\, dy$$

4. (12 points) Find the volume of the space region bounded above by the cone $z^2 = x^2+y^2$ and below by the region that lies inside the curve $x^2 + y^2 = 2ax$, where a represents a positive real number. Carry out the integration by hand, but you may use a CAS to check your work.

5. (16 points) A thin plate has the shape of a triangle with vertices at $(0,0)$, $(1,1)$ and $(2,0)$. The density of the plate at the point (x,y) is given by $\rho(x,y) = 2x + y + 1$. Find the center of mass of the plate. Compute the mass by hand, but you may use a CAS for the moments.

Math 173 - Test 3b

May 3, 2016

Name ____________________

Score ____________

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Sketch the region of integration and evaluate $\displaystyle\int_0^1 \int_y^1 \frac{e^x}{x}\, dx\, dy$.

   (Hint: Reverse the order of integration.)

2. (10 points) Use the method of Lagrange multipliers to find the point on the plane $x + 2y + 3z = 6$ that is closest to the origin. (Hint: Minimize $f(x, y, z) = x^2 + y^2 + z^2$, which gives the square of the distance from the origin.)

3. (10 points) Find the average value of $f(x, y) = x^2 + y^2$ over the region bounded by the graphs of $y = x^2$ and $x = y^2$.

4. (10 points) Let S be the space region bounded by the surfaces $x = 0$, $y = 0$, $y = 1 - x$, and $z = 1 - x^2$ (see the figure below). Evaluate the triple integral $\displaystyle\iiint\limits_S z\, dV$.