

Math 173 - Test 4
May 16, 2016

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) Let $\vec{u} = 3\hat{i} - 5\hat{k}$ and $\vec{w} = -\hat{i} + 2\hat{j} + 3\hat{k}$.

(a) Find a vector of magnitude 2 that has the direction of \vec{w} .

(b) Find the measure of the angle between \vec{u} and \vec{w} .

(c) Find the projection of \vec{w} onto \vec{u} .

2. (10 points) Write a direction vector for each line. Then show that the lines are parallel.

$$L_1 : \quad x = 2 + 3t, \quad y = -5 - t, \quad z = 8t$$

$$L_2 : \quad \frac{x+2}{6} = \frac{1-y}{2} = \frac{z+7}{16}$$

3. (10 points) Find an equation of the plane passing through the points $P(0, 2, 1)$, $Q(2, -1, 3)$, and $R(5, 0, 8)$.

4. (10 points) Let $f(x, y, z) = x^2yz^3$. Find the gradient vector and the maximum directional derivative at the point $(1, 2, 1)$.

5. (10 points) Find the critical points of $g(x, y) = x^3 + y^3 - 3xy + 9$. Then use the 2nd partials test to classify the critical points and determine the relative extrema.

6. (10 points) A baseball is hit from 2.8 ft above home plate with an initial velocity vector of $82\hat{i} + 78\hat{j}$ (measured in ft/s). Will the ball clear a 20-ft wall 380 ft from home plate. Neglect all forces except gravity and use $g = 32 \text{ ft/s}^2$.

7. (12 points) A thin plate of constant density ρ lies in the 1st quadrant bounded by the graphs of $y = (x - 1)^2$ and $y = x + 1$. Write the double integral that gives the moment about the y -axis. Then sketch the region of integration, write the integral as an iterated integral, and evaluate (by hand).

8. (12 points) Determine the limit or show that it does not exist.

$$(a) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{\sqrt{x^2 + y^2}}$$

$$(b) \lim_{(x,y) \rightarrow (0,0)} \frac{3xy^2}{x^2 + y^4}$$

9. (10 points) The position of a particle at time t is given by $\vec{r}(t) = e^t \hat{i} + 2e^{-t} \hat{j} + t \hat{k}$. Write the definite integral that gives the length of the particle's path from $t = 0$ to $t = \ln 3$. Use your calculator to approximate the value of the integral.

10. (12 points) Consider the vector field

$$\vec{F}(x, y, z) = (y + y^2z)\hat{i} + (x - z + 2xyz)\hat{j} + (-y + xy^2)\hat{k}.$$

(a) Find a scalar potential function $f(x, y, z)$ for $\vec{F}(x, y, z)$.

(b) Use any method to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the line segment joining $(2, 2, 1)$ and $(1, -1, 2)$.

11. (8 points) Write the iterated integral with the reversed order of integration. Do not evaluate.

$$\int_0^2 \int_{x^3}^{4x} x^2 dy dx$$

12. (10 points) Find an equation of the plane tangent to the graph of $f(x, y) = \frac{5}{x^2 + y^2}$ at the point where $(x, y) = (-1, 2)$.

13. (12 points) Use the method of Lagrange multipliers to find the extreme values of $f(x, y) = 2x^2 + y^2 + 5$ subject to the constraint $x^2 + 4y^2 = 4$.

14. (12 points) Convert to cylindrical coordinates and evaluate.

$$\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^2 \frac{1}{1+x^2+y^2} dz dy dx$$