

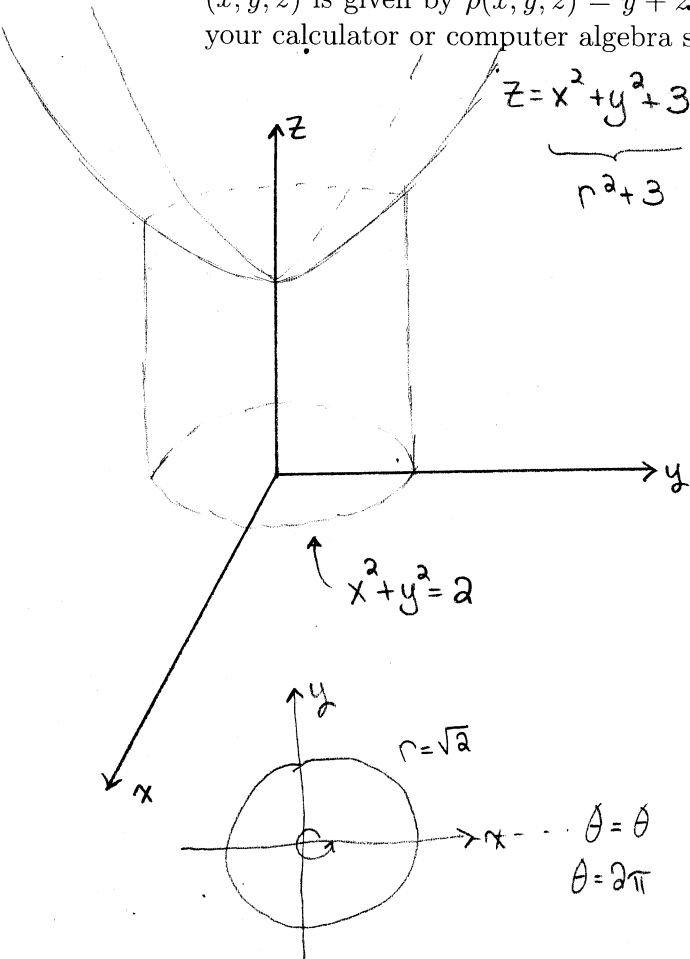
# Math 173 - Quiz 11

May 4, 2017

Name key Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) The solid region inside the cylinder  $x^2 + y^2 = 2$  is bounded below by the surface  $z = 0$  and above by  $z = x^2 + y^2 + 3$ . The density of the solid at the point  $(x, y, z)$  is given by  $\rho(x, y, z) = y + z^2 + 1$ . Find the center of mass of the solid. Use your calculator or computer algebra system to evaluate all required integrals.



$$\begin{aligned}
 \text{Mass} &= \iiint_S \rho(x, y, z) \, dV \\
 &= \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{2}} \int_{z=0}^{r^2+3} (r \sin \theta + z^2 + 1) r \, dz \, dr \, d\theta \\
 &= \frac{160\pi}{3}
 \end{aligned}$$

$$\begin{aligned}
 M_{yz} &= \iiint_S x \rho(x, y, z) \, dV \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 M_{xz} &= \iiint_S y \rho(x, y, z) \, dV \\
 &= \frac{13\pi}{3}
 \end{aligned}$$

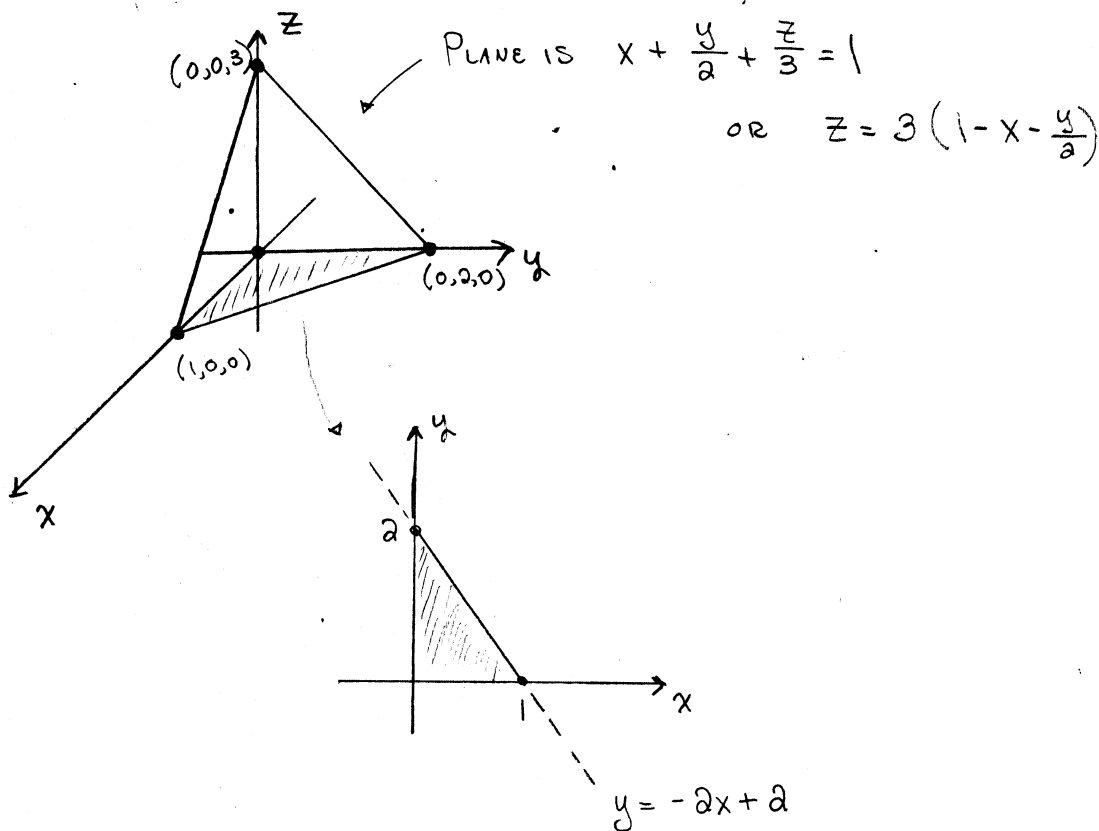
$$\begin{aligned}
 M_{xy} &= \iiint_S z \rho(x, y, z) \, dV \\
 &= \frac{4813\pi}{30}
 \end{aligned}$$

CONVERT TO CYLINDRICAL  
COORDS...

$$\begin{aligned}
 x &= r \cos \theta \\
 y &= r \sin \theta \\
 x^2 + y^2 &= r^2
 \end{aligned}$$

CM is  $\left( 0, \frac{13}{160}, \frac{4813}{1600} \right)$

2. (5 points) Let  $T$  be the tetrahedron in the 1st octant whose vertices are  $(0,0,0)$ ,  $(1,0,0)$ ,  $(0,2,0)$ , and  $(0,0,3)$ . Find the average value of  $f(x,y,z) = x + y^2 + z^3$  on  $T$ . Use your calculator or computer algebra system to evaluate all required integrals.



$$\text{Volume} = \int_{x=0}^1 \int_{y=0}^{-2x+2} \int_{z=0}^{3\left(1-x-\frac{y}{2}\right)} dz dy dx = 1$$

$$\begin{aligned} \text{AVERAGE VALUE} &= \frac{1}{\text{Vol}} \int_{x=0}^1 \int_{y=0}^{-2x+2} \int_{z=0}^{3\left(1-x-\frac{y}{2}\right)} (x + y^2 + z^3) dz dy dx \\ &= \boxed{2} \end{aligned}$$