Math 173 - Quiz 2

January 26, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the angle between the vectors $\vec{u} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$ and $\vec{v} = 4\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \frac{8-2-4}{\sqrt{21}\sqrt{21}} = \frac{2}{21}$$

$$\theta = \cos^{-1}\left(\frac{2}{21}\right) \approx 1.475 \approx 84.53^\circ$$

2. (3 points) Let $\vec{u} = \hat{\imath} + \hat{\jmath} - 5\hat{k}$. Find a unit vector that is orthogonal to $\vec{u}$.

We need $\vec{\omega}$ so that $\vec{\omega}\cdot\vec{u} = 0 = \omega_1 + \omega_2 - 5\omega_3$.

I choose $\omega_1 = 1$, $\omega_2 = -1$, $\omega_3 = 0$.

$$\vec{\omega} = \hat{\imath} - \hat{\jmath}$$

$$\frac{\vec{\omega}}{\|\vec{\omega}\|} = \frac{1}{\sqrt{2}}(\hat{\imath} - \hat{\jmath})$$

3. (2 points) Let $\vec{u} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$ and $\vec{v} = 4\hat{\imath} - 2\hat{\jmath} + \hat{k}$. Compute $\vec{u} \times \vec{v}$.

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -4 \\ 4 & -2 & 1 \end{vmatrix} = \hat{\imath}(1-8) - \hat{\jmath}(2+16) + \hat{k}(-4-4)$$

$$= -7\hat{\imath} - 18\hat{\jmath} - 8\hat{k}$$

4. (2 points) Show that $\|\text{proj}_{\vec{w}}\vec{u}\|\,\|\vec{w}\| = |\vec{u}\cdot\vec{w}|$.

$$\text{proj}_{\vec{w}}\,\vec{u} = \frac{\vec{u}\cdot\vec{w}}{\|\vec{w}\|^2}\,\vec{w}$$

$$\|\text{proj}_{\vec{w}}\,\vec{u}\|\,\|\vec{w}\| = \left\|\frac{\vec{u}\cdot\vec{w}}{\|\vec{w}\|^2}\,\vec{w}\right\|\|\vec{w}\|$$

$$= |\vec{u}\cdot\vec{w}|\left\|\frac{1}{\|\vec{w}\|^2}\,\vec{w}\right\|\|\vec{w}\| = |\vec{u}\cdot\vec{w}|$$