

Math 173 - Quiz 3

February 9, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3.5 points) Find the area of the triangle with vertices $P(1, 2, 3)$, $Q(2, 5, -4)$, and $R(-1, -3, 4)$.

$$\begin{aligned} \vec{PQ} &= \hat{i} + 3\hat{j} - 7\hat{k} \\ \vec{PR} &= -2\hat{i} - 5\hat{j} + \hat{k} \end{aligned}$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -7 \\ -2 & -5 & 1 \end{vmatrix} = \hat{i}(-32) - \hat{j}(-13) + \hat{k}(1)$$

$$\text{Area} = \frac{1}{2} \|\vec{PQ} \times \vec{PR}\| = \frac{1}{2} \sqrt{(32)^2 + (13)^2 + 1^2} = \frac{1}{2} \sqrt{597} \approx 17.2772$$

2. (1 point) Find an equation for the plane passing through the origin and parallel to the plane containing the triangle above.

$$\vec{n} = -32\hat{i} + 13\hat{j} + \hat{k}$$

POINT (0, 0, 0)

$$\Rightarrow -32x + 13y + z = 0$$

3. (2 points) Find a set of parametric equations for the line passing through $(8, -4, 2)$ and parallel to the line with symmetric equations

$$\frac{x+3}{5} = \frac{y-2}{-3} = z+1$$

$$\vec{v} = 5\hat{i} - 3\hat{j} + \hat{k}$$

POINT (8, -4, 2)

$$\Rightarrow x = 8 + 5t, y = -4 - 3t, z = 2 + t$$

4. (3.5 points) Find a set of parametric equations for the line of intersection of the following planes

$$\begin{aligned} \vec{n}_1 &= 2\hat{i} + \hat{j} - \hat{k} \\ \vec{n}_2 &= 3\hat{i} + 2\hat{j} + \hat{k} \end{aligned}$$

$$\vec{u} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix} = \hat{i}(3) - \hat{j}(5) + \hat{k}(1) = 3\hat{i} - 5\hat{j} + \hat{k}$$

$$2x + y - z = 4$$

$$3x + 2y + z = 8$$

↑ TO FIND A PT ON LINE OF INTERSECTION,

LET $x=0$: $y - z = 4$
 $2y + z = 8$

 $3y = 12$
 $y = 4, z = 0$

POINT (0, 4, 0)

$$\Rightarrow x = 3t, y = 4 - 5t, z = t$$