Math 173 - Quiz 4

February 16, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Find $\vec{r}(t)$ if $\vec{r}'(t) = e^{2t}\hat{\imath} + \sec^2(t)\hat{\jmath} + (t^2+3)\hat{k}$ and $\vec{r}(0) = 3\hat{\imath} + \hat{\jmath} - 4\hat{k}$.

$$\vec{r}(t) = \left(\tfrac{1}{2}e^{2t} + C_1\right)\hat{\imath} + \left(\tan t + C_2\right)\hat{\jmath} + \left(\tfrac{1}{3}t^3 + 3t + C_3\right)\hat{k}$$

$$\vec{r}(0) = \left(\tfrac{1}{2} + C_1\right)\hat{\imath} + C_2\hat{\jmath} + C_3\hat{k} = 3\hat{\imath} + \hat{\jmath} - 4\hat{k}$$

$$\Rightarrow C_1 = \tfrac{5}{2},\ C_2 = 1,\ C_3 = -4$$

$$\vec{r}(t) = \left(\tfrac{1}{2}e^{2t} + \tfrac{5}{2}\right)\hat{\imath} + \left(\tan t + 1\right)\hat{\jmath} + \left(\tfrac{1}{3}t^3 + 3t - 4\right)\hat{k}$$

2. (4 points) Let $\vec{u}(t) = t\hat{\imath} + 3t\hat{\jmath} + t^2\hat{k}$ and $\vec{w}(t) = 4t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}$. Compute $\frac{d}{dt}[\vec{u}(t) \times \vec{w}(t)]$.

$$\vec{u}\times\vec{w} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ t & 3t & t^2 \\ 4t & t^2 & t^3 \end{vmatrix} = \hat{\imath}\left(3t^4 - t^4\right) - \hat{\jmath}\left(t^4 - 4t^3\right) + \hat{k}\left(t^3 - 12t^2\right)$$

$$= 2t^4\hat{\imath} + \left(4t^3 - t^4\right)\hat{\jmath} + \left(t^3 - 12t^2\right)\hat{k}$$

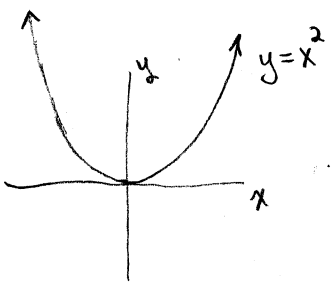
$$\frac{d}{dt}\left[\vec{u}\times\vec{w}\right] = 8t^3\hat{\imath} + \left(12t^2 - 4t^3\right)\hat{\jmath} + \left(3t^2 - 24t\right)\hat{k}$$

3. (2 points) Describe the graphs of the vector-valued functions $\vec{r}_1(t) = t\hat{\imath} + t^2\hat{\jmath}$ and $\vec{r}_2(t) = t\hat{\imath} + t^2\hat{\jmath} + t\hat{k}$.

$\vec{r}_1(t) = t\hat{\imath} + t^2\hat{\jmath}$

$x = t$, $y = t^2 \Rightarrow y = x^2$

GRAPH OF $\vec{r}_1(t)$ IS THE $y = x^2$ PARABOLA.



GRAPH OF $\vec{r}_2(t)$ IS THE SAME PARABOLA BUT TWISTED 45° OUT OF THE xy-PLANE SO THAT THE GRAPH LIES IN THE $x = z$ PLANE.