

Math 173 - Quiz 6

March 30, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the directional derivative of $g(x, y) = \sqrt{x^2 + y^2}$ at the point $(3, 4)$ in the direction of $3\hat{i} - 4\hat{j}$.

$$\vec{\nabla}g(x, y) = \frac{1}{2}(x^2 + y^2)^{-1/2} (2x\hat{i} + 2y\hat{j})$$

$$\vec{\nabla}g(3, 4) = \frac{3\hat{i} + 4\hat{j}}{5}$$

$$\vec{v} = 3\hat{i} - 4\hat{j}$$

$$\|\vec{v}\| = 5$$

$$\frac{\vec{\nabla}g(3, 4) \cdot \vec{v}}{\|\vec{v}\|} = \frac{9 - 16}{25} = \boxed{\frac{-7}{25}}$$

2. (3 points) Let $f(x, y, z) = x \tan(y + z)$. Compute the gradient of f at the point $(4, 3, -1)$.

$$\vec{\nabla}f(x, y, z) = \tan(y + z)\hat{i} + x \sec^2(y + z)\hat{j} + x \sec^2(y + z)\hat{k}$$

$$\vec{\nabla}f(4, 3, -1) = \tan(a)\hat{i} + 4 \sec^2(a)\hat{j} + 4 \sec^2(a)\hat{k}$$

3. (4 points) Let $f(x, y) = x^2y + 2xy^3$. Find a unit vector that is normal to the level curve through the point $(3, 2)$.

$$\vec{\nabla}f(x, y) = (2xy + 2y^3)\hat{i} + (x^2 + 6xy^2)\hat{j}$$

$$\vec{\nabla}f(3, 2) = 28\hat{i} + 81\hat{j}$$

$$\|\vec{\nabla}f(3, 2)\| = \sqrt{28^2 + 81^2} = \sqrt{7345}$$

$$\frac{28\hat{i} + 81\hat{j}}{\sqrt{7345}}$$