

Math 173 - Quiz 7

April 5, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Consider the limit

$$\lim_{(x,y) \rightarrow (2,1)} \frac{x^2 + 4y^2 - 4xy + 7x - 14y}{5x - 10y} \quad \frac{0}{0}$$

Notice that substitution gives rise to an indeterminate form. Evaluate the limit by replacing the numerator with its linearization at (2,1).

$$f(x,y) = x^2 + 4y^2 - 4xy + 7x - 14y, \quad f(2,1) = 0$$

$$f_x(x,y) = 2x - 4y + 7, \quad f_x(2,1) = 7$$

$$f_y(x,y) = 8y - 4x - 14, \quad f_y(2,1) = -14$$

$$L(x,y) = 0 + 7(x-2) - 14(y-1) \\ = 7x - 14y$$

$$\lim_{(x,y) \rightarrow (2,1)} \frac{7x - 14y}{5x - 10y} \\ = \lim_{(x,y) \rightarrow (2,1)} \frac{7(x-2y)}{5(x-2y)} \\ = \frac{7}{5}$$

2. (7 points) Find and classify the critical points $f(x,y) = y^4 - 4xy + x^2 + 8$.

$$f_x(x,y) = -4y + 2x = 0 \Rightarrow x = 2y$$

$$f_y(x,y) = 4y^3 - 4x = 0$$

$$4y^3 - 8y = 0$$

$$4y(y^2 - 2) = 0$$

$$y = 0 \quad y = \pm\sqrt{2}$$

$$x = 0 \quad x = \pm 2\sqrt{2}$$

$$(0,0), (2\sqrt{2}, \sqrt{2})$$

$$(-2\sqrt{2}, -\sqrt{2})$$

$$d(x,y) = \begin{vmatrix} 2 & -4 \\ -4 & 12y^2 \end{vmatrix} = 24y^2 - 16$$

$$d(0,0) = -16 \Rightarrow (0,0,8) \text{ IS A SAADDLE PT.}$$

$$d(2\sqrt{2}, \sqrt{2}) > 0 \ \& \ f_{xx}(2\sqrt{2}, \sqrt{2}) > 0$$

$$\Rightarrow f(2\sqrt{2}, \sqrt{2}) = 4 \text{ IS A REL MIN.}$$

$$d(-2\sqrt{2}, -\sqrt{2}) > 0 \ \&$$

$$f_{xx}(-2\sqrt{2}, -\sqrt{2}) > 0$$

$$\Rightarrow f(-2\sqrt{2}, -\sqrt{2}) = 4 \text{ IS A REL MIN.}$$