Math 173 - Quiz 8

April 5, 2017

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Assume that $x^3e^{xz} + \ln(xyz) = 8$ implicitly defines z as a function of x and y. Find $\partial z/\partial x$.

$$F(x,y,z) = x^3e^{xz} + \ln(xyz)$$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = \frac{-\left(3x^2e^{xz} + x^3ze^{xz} + \frac{1}{x}\right)}{x^4e^{xz} + \frac{1}{z}}$$

2. (3 points) Let $f(x,y) = \sin(2xy) + xe^{-2xy}$. Find an equation of the plane tangent to the graph of f at the point where $(x,y) = (\pi, 0)$. $f(\pi,0) = \pi$

$$z = \sin(2xy) + xe^{-2xy} \Rightarrow \underbrace{\sin(2xy) + xe^{-2xy} - z}_{F(x,y,z)} = 0$$

OUR SURFACE IS THE LEVEL SURFACE

$$F(x,y,z) = 0$$

$$\vec{\nabla} F(x,y,z) = \left(2y\cos(2xy) + e^{-2xy} - 2xye^{-2xy}\right)\hat{i} + \left(2x\cos(2xy) - 2x^2e^{-2xy}\right)\hat{j} - \hat{k}$$

$$\vec{n} = \vec{\nabla} F(\pi,0,\pi) = \hat{i} + (2\pi - 2\pi^2)\hat{j} - \hat{k}$$

TANGENT PLANE:

$$(x-\pi) + (2\pi - 2\pi^2)(y-0) - (z-\pi) = 0$$

-OR-

$$x + (2\pi - 2\pi^2)y - z = 0$$

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

3. (5 points) Suppose we are given the two functions

$$f(x,y) = 2x^2y + x - 5y - 7, \qquad g(x,y) = xy^2 + 2x - y - 6$$

and we wish to solve the system of nonlinear equations $f(x,y) = 0$, $g(x,y) = 0$. We can approximate a solution using linearizations and Newton's method.

(a) Let $(x_0, y_0) = (1, -2)$ be our initial guess at the solution. Let $f_0(x,y)$ and $g_0(x,y)$ be the linearizations of f and g at the point (x_0, y_0). Find $f_0(x,y)$ and $g_0(x,y)$.

$f_x(x,y) = 4xy + 1$

$f_y(x,y) = 2x^2 - 5$

$g_x(x,y) = y^2 + 2$

$g_y(x,y) = 2xy - 1$

$f_0(x,y) = f(1,-2) + f_x(1,-2)(x-1) + f_y(1,-2)(y+2)$

$= -7(x-1) + (-3)(y+2)$

$= -7x - 3y + 1$

$g_0(x,y) = g(1,-2) + g_x(1,-2)(x-1) + g_y(1,-2)(y+2)$

$= 2 + 6(x-1) + (-5)(y+2)$

$= 6x - 5y - 14$

(b) Solve the linear system of equations

$$f_0(x,y) = 0, \qquad g_0(x,y) = 0.$$

$7x + 3y = 1 \Rightarrow 35x + 15y = 5$

$6x - 5y = 14 \Rightarrow 18x - 15y = 42$

$53x = 47$

$x = \frac{47}{53}, \quad y = \frac{1-7x}{3} = \frac{-92}{53}$

(c) Let (x_1, y_1) be the solution of the linear system in part (b). It represents an improved guess at the solution of the original system. Compute $f(x_1, y_1)$ and $g(x_1, y_1)$.

$f\left(\frac{47}{53}, -\frac{92}{53}\right) = -\frac{24432}{148877} \approx -0.164$

$g\left(\frac{47}{53}, -\frac{92}{53}\right) = \frac{27020}{148877} \approx 0.181$

(d) (Extra Credit 3 pts) On a separate sheet of paper, use (x_1, y_1) in place of (x_0, y_0) and repeat the steps above to further improve our guess at the solution. You may use a computer algebra system to do the computations.

SEE ATTACHED SHEET.

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(%i2)   f:2*x^2*y+x-5*y-7;
        g:x*y^2+2*x-y-6;
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(f) $2\,x^2\,y-5\,y+x-7$

(g) $x\,y^2-y+2\,x-6$

```
(%i6)   fx:diff(f,x);
        fy:diff(f,y);
        gx:diff(g,x);
        gy:diff(g,y);
```

(fx) $4\,x\,y+1$

(fy) $2\,x^2-5$

(gx) y^2+2

(gy) $2\,x\,y-1$

```
(%i12)  x0:1;
        y0:-2;
        L1:at(f,[x=x0,y=y0])+at(fx,[x=x0,y=y0])*(x-x0)+at(fy,[x=x0,y=y0])*(y-y
        L2:at(g,[x=x0,y=y0])+at(gx,[x=x0,y=y0])*(x-x0)+at(gy,[x=x0,y=y0])*(y-y
        solve([L1=0,L2=0],[x,y]);
        float(%);
```

(x0) 1

(y0) -2

(L1) $-3\,(y+2)-7\,(x-1)$

(L2) $-5\,(y+2)+6\,(x-1)+2$

(%o11) $[[x=\frac{47}{53},y=-\frac{92}{53}]]$

(%o12) $[[x=0.8867924528301887,y=-1.735849056603774]]$

```
(%i18)  x0:47/53;
        y0:-92/53;
        L1:at(f,[x=x0,y=y0])+at(fx,[x=x0,y=y0])*(x-x0)+at(fy,[x=x0,y=y0])*(y-y
        L2:at(g,[x=x0,y=y0])+at(gx,[x=x0,y=y0])*(x-x0)+at(gy,[x=x0,y=y0])*(y-y
        solve([L1=0,L2=0],[x,y]);
        float(%);
```

(x0) $\frac{47}{53}$

(y0) $-\frac{92}{53}$

(L1) $-\frac{9627\left(y+\frac{92}{53}\right)}{2809}-\frac{14487\left(x-\frac{47}{53}\right)}{2809}-\frac{24432}{148877}$

(L2) $-\frac{11457\left(y+\frac{92}{53}\right)}{2809}+\frac{14082\left(x-\frac{47}{53}\right)}{2809}+\frac{27020}{148877}$

(%o17) $[[x=\frac{4544191589}{5327294523},y=-\frac{9231583400}{5327294523}]]$

(%o18) $[[x=0.8530017571547733,y=-1.732883992079595]]$