

# Math 173 - Quiz 9

April 13, 2017

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (10 points) Find the maximum and minimum values of  $f(x, y) = xy$  on the unit circle  $x^2 + y^2 = 1$ .

$$f(x, y) = xy$$

$$g(x, y) = x^2 + y^2$$

$$\vec{\nabla} f = \lambda \vec{\nabla} g$$

$$g = 1$$

$$y = \lambda 2x$$

$$x = \lambda 2y$$

$$x^2 + y^2 = 1$$

$$2\lambda x = 4\lambda^2 y$$

$$y = 4\lambda^2 y$$

$$4\lambda^2 y - y = 0$$

$$y(4\lambda^2 - 1) = 0$$

$$y = 0 \quad \text{or} \quad \lambda = \pm \frac{1}{2}$$

$$\downarrow$$

$$x = 0$$

$$\downarrow$$

$$y = x, -x$$

$$x^2 + x^2 = 1$$

$\Downarrow$

$$x = \pm \frac{1}{\sqrt{2}}$$

BUT  
(0,0)  
DOESN'T  
SATISFY  
 $x^2 + y^2 = 1$

$$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

$$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

$$f\left(\pm \frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}}\right) = \frac{1}{2} \quad \text{MAX}$$

$$f\left(\pm \frac{1}{\sqrt{2}}, \mp \frac{1}{\sqrt{2}}\right) = -\frac{1}{2} \quad \text{MIN}$$