Math 173 - Test 1

February 23, 2017

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) The vector $\vec{u}$ is the 2D vector that has magnitude 5 and makes a $30°$ angle with the positive x-axis. The vector $\vec{v}$ is the 2D vector that has magnitude 3 and makes a $90°$ angle with the positive x-axis. Compute $\vec{u} + \vec{v}$. Then sketch $\vec{u}$, $\vec{v}$, and $\vec{u} + \vec{v}$ in the xy-plane and indicate how these vectors are related by the parallelogram law.

2. (6 points) Use vectors to show that the points A, B, and C are collinear.

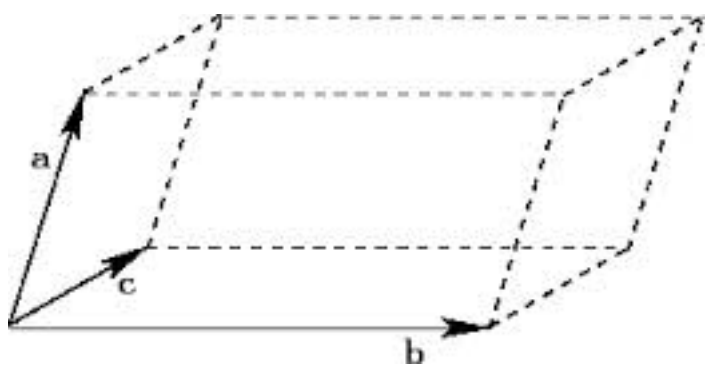
$$A\,(-7, 9, 10) \qquad B\,(3, 4, -5) \qquad C\,(1, 5, -2)$$

3. (4 points) Sketch a diagram that shows two vectors, $\vec{v}$ and $\vec{w}$, and then show the vector $\text{proj}_{\vec{v}}\,\vec{w}$.

4. (4 points) What does it mean for two vectors to be orthogonal? Use your answer to show that $\vec{u} = 3\hat{\imath} - 7\hat{\jmath} + \hat{k}$ is orthogonal to $\vec{v} = -5\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

5. (4 points) If the projection of $\vec{u}$ onto $\vec{v}$ has the same magnitude as the projection of $\vec{v}$ onto $\vec{u}$, can you conclude that $\|\vec{u}\| = \|\vec{v}\|$? Explain.

6. (8 points) A crystal structure has the form of a parallelepiped determined by the vectors $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = -\hat{\imath} + \hat{\jmath} - 4\hat{k}$, and $\vec{c} = 4\hat{\imath} + 2\hat{\jmath} - \hat{k}$, where distances are measured in micrometers. Find the volume of the parallelepiped.



7. (12 points) The points $P\,(0,2,3)$, $Q\,(-1,2,4)$, and $R\,(3,-7,2)$ are the vertices of a triangle.

   (a) Find the area of the triangle.

   (b) Find an equation of the plane containing the triangle.

   (c) Find a set of parametric equations for the line segment $\overline{PR}$.

8. (5 points) Describe (or sketch) the 3D surface defined by the equation $z = e^x$.

9. (7 points) Find the vector-valued function $\vec{r}(t)$ such that

$$\vec{r}\,'(t) = te^{-t^2}\hat{\imath} - e^{-t}\hat{\jmath} + \hat{k}; \quad \vec{r}(0) = \frac{1}{2}\hat{\imath} - \hat{\jmath} + 2\hat{k}.$$

10. (10 points) A projectile is fired from ground level at an angle of 8° with the horizontal. The projectile is to have a range of 50 meters. Find the required initial velocity and the maximum height of the projectile.

11. (1 point) If you were given two nonparallel vectors, how could you find a nonzero vector orthogonal to both?

12. (4 points) What is the domain of the function $\vec{r}(t) = \sqrt{t}\,\hat{\imath} + \tan t\,\hat{\jmath} + \ln(t-1)\,\hat{k}$?

13. (3 points) Refer to the function above. Is $\vec{r}(t)$ continuous at $t = 4$? Explain.

14. (3 points) Suppose $\vec{r}(t)$ describes a line in space. What can be said about $\hat{T}'(t)$? Explain.

15. (6 points) Let $\vec{r}(t) = t\hat{\imath} + t^2\hat{\jmath} + \ln(t)\hat{k}$. Compute $\hat{T}(1)$.

16. (8 points) For $t \geq 0$, sketch the graph of $\vec{r}(t) = t^3\hat{\imath} + t\hat{\jmath}$. Without computing them, sketch the unit tangent vector and the principal unit normal vector at the point where $t = 1$.

17. (3 points) The angle between $\vec{u}$ and $\vec{v}$ is obtuse. What can be said about $\vec{u} \cdot \vec{v}$? Briefly explain.

18. (4 points) Let A, B, and C be the vertices of a triangle. Determine $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$.