

**Math 173 - Test 2**  
March 23, 2017

Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations where necessary.

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1. (3 points) Describe the graph of a vector-valued function for which the curvature is 0 for all values of  $t$  in its domain.

2. (4 points) Let  $g(x, y) = \int_x^y (2t - 3) dt$ . Evaluate  $g(4, 1)$ .

3. (4 points) Suppose  $w$  is a function of  $x, y, z$ ; and  $x, y, z$  are functions of  $s, t$ . Write the chain rule formula for  $\frac{\partial w}{\partial t}$ .

4. (3 points) Describe the graph of a vector-valued function for which the curvature is 1 for all values of  $t$  in its domain.

5. (10 points) Consider the function  $f(x, y) = \ln(4 - x - y)$ .

(a) What is the domain of  $f$ ?

(b) What is the range of  $f$ ?

(c) Sketch the level curve  $f(x, y) = 0$ .

(d) Sketch the level curve  $f(x, y) = 1$ .

(e) At which points is  $f$  continuous?

6. (12 points) Determine each limit or explain why the limit does not exist.

$$(a) \lim_{(x,y) \rightarrow (4,1)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$$

$$(b) \lim_{(x,y) \rightarrow (1,1)} \frac{x - y}{x + y - 2}$$

$$(c) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^2}$$

7. (8 points) Suppose  $w = f(x, y)$ , where  $x = u - v$  and  $y = v - u$ . Use the chain rule to show that  $\frac{\partial w}{\partial u} + \frac{\partial w}{\partial v} = 0$ .

8. (12 points) State whether the graph of the equation is an ellipsoid, a paraboloid, a cone, a hyperboloid of one sheet, a hyperboloid of two sheets, or a hyperbolic paraboloid.

(a)  $4x^2 - 3y^2 + 12z^2 + 12 = 0$

(b)  $x - y^2 - 4z^2 = 0$

(c)  $\frac{x^2}{7} + 4y^2 + z^2 = 1$

(d)  $z = \frac{y^2}{16} - \frac{x^2}{16}$

(e)  $x^2 = 2y^2 + 4z^2$

(f)  $y^2 + 2 = x^2 + 5z^2$

9. (4 points) Sketch the graph of the function  $f(x, y) = -x^2 - y^2$ .

10. (9 points) Suppose  $z = 2xe^{5y} - 3ye^{-x}$ .

(a) Which first partial derivative should be computed first in order to obtain  $\frac{\partial^2 z}{\partial x \partial y}$ ?

(b) Do you expect to have  $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ ? Explain.

(c) Compute  $\frac{\partial^2 z}{\partial x \partial y}$ .

11. (10 points) The *body mass index* (BMI) for an adult human is given by  $B = 703w/h^2$ , where  $w$  is weight in pounds and  $h$  is height in inches. Suppose you weigh 185 lbs and your height is 68 in. Compute your BMI. Then assume your weight and height measurements have errors  $\Delta w = 1.75$  lbs and  $\Delta h = 0.5$  in. Use differentials to estimate the error in your BMI.

12. (8 points) Sketch the plane curve and find its length over the given interval. You may use your calculator to approximate the value of the integral. (Hint: It might help to eliminate the parameter in order to sketch the graph.)

$$\vec{r}(t) = (t + 1)\hat{i} + t^2\hat{j}, \quad [0, 6]$$

13. (8 points) Find the curvature at the point  $P$ .

$$\vec{r}(t) = t\hat{i} + t^2\hat{j} + \frac{t^3}{4}\hat{k}, \quad P(2, 4, 2)$$

14. (5 points) Find all values of  $x$  and  $y$  where  $f_x(x, y) = 0$  and  $f_y(x, y) = 0$  simultaneously.

$$f(x, y) = x^2 + 4xy + y^2 - 4x + 16y + 3$$