Math 173 - Final Exam

May 15, 2017

Name ____________

Score ____________

Show all work to receive full credit. Supply explanations where necessary.

1. (15 points) Let P and Q be the points $(2, 1, -3)$ and $(4, -2, 7)$, respectively.

   (a) Find a vector of length 2 in the direction of $\overrightarrow{PQ}$.

   (b) Find a unit vector in the xy-plane that is orthogonal to $\overrightarrow{PQ}$.

   (c) Find the projection of $\overrightarrow{PQ}$ onto $3\hat{\imath} + 2\hat{\jmath} + \hat{k}$.

2. (10 points) Consider the plane described by the equation $5x - 3y - 2z = 8$.

   (a) Find an equation of the parallel plane passing through the point $(2, 3, 5)$.

   (b) Find the angle between the given plane and the plane with equation $2x + 7y = 12$.

3. (10 points) A plane passes through the points $P\,(2, 1, 3)$, $Q\,(-7, 6, -1)$ and $R\,(3, 0, -1)$. Find a set of parametric equations for the line normal to the plane and passing through $(2, 1, 3)$.

4. (10 points) The quarterback of a football team throws the ball with an initial speed of 54 feet per second, at an angle of $36°$, and at a height of 6 feet above the playing field. How far downfield has the ball traveled at the moment when it reaches its maximum height? (Ignore air resistance and use $g = 32\,\mathrm{ft}/\mathrm{sec}^2$.)

5. (15 points) Let $\vec{r}(t) = 3t\hat{\imath} + \sin 2t\,\hat{\jmath} + \cos 2t\,\hat{k}$. Find (a) the unit tangent vector, (b) the principal unit normal vector, and (c) the length of the graph from $t = 0$ to $t = \pi/2$.

6. (10 points) Suppose that $w = 3xy + yz$ and that x, y, and z are functions of u and v such that

$$x = \ln u + \cos v, \qquad y = 1 + u \sin v, \qquad z = uv.$$

Use the chain rule to find $\partial w/\partial u$ at $(u, v) = (1, \pi)$.

7. (10 points) The temperature at the point (x, y) on a ceramic plate is given by

$$T(x, y) = 12 + \cos(\pi xy) + x^2 + xy, \quad -2 \leq x \leq 2, \ -2 \leq y \leq 2.$$

(a) Find the directional derivative of T at the point $(1, 1)$ in the direction of $3\hat{\imath} + \hat{\jmath}$.

(b) Find the direction of greatest temperature decrease at the point $(1, 1)$.

8. (10 points) Consider the surface in 3-space defined by the equation $x^2yz^3 = 2$. Find an equation of the plane tangent to the surface at the point $(1, 2, 1)$.

9. (10 points) Use Lagrange multipliers to find the minimum and maximum values of $f(x, y, z) = 2x + 3y + z$ on the sphere $x^2 + y^2 + z^2 = 1$.

10. (12 points) Sketch the region of integration, reverse the order of integration, and **evaluate the iterated integral by hand**.

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{x}{y^5+1}\, dy\, dx$$

11. (12 points) Use a triple integral to find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$. (Evaluate the integral by hand.)

12. (13 points) Convert to cylindrical coordinates and evaluate by hand.

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} \int_{0}^{3} \frac{1}{1+x^2+y^2}\, dz\, dy\, dx$$

13. (13 points) Show that $\vec{F}(x,y) = \frac{1}{2}xy\hat{\imath} + \frac{1}{4}x^2\hat{\jmath}$ is a conservative vector field. Then use any method to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is any smooth curve from $(0,0)$ to $(1,1)$. Evaluate your integral by hand.