Math 173 - Test 1

February 15, 2018

Name _____

Score _____

Show all work. Supply explanations when necessary.

1. (4 points) Let $\vec{w} = -3\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector of magnitude 6 in the direction opposite of $\vec{w}$.

2. (8 points) Let P and Q be the points $(3, 5, -2)$ and $(5, 3, -6)$, respectively.

   (a) Find the midpoint of the line segment connecting P and Q.

   (b) Find a nonzero vector that is orthogonal to $\vec{PQ}$.

   (c) Find a set of parametric equations for the line passing through the midpoint of segment PQ and parallel to the vector you found in part (b).

3. (6 points) Forces with magnitudes of 500 lb and 200 lb act on a machine part at angles of $30°$ and $-45°$, respectively, with the positive x-axis. Find the magnitude of the resultant force.

4. (12 points) Consider the triangle with vertices $A(5, 3, 1)$, $B(3, 2, 3)$, and $C(-4, -1, 2)$.

   (a) Compute $\vec{AB} \times \vec{AC}$.

   (b) Find the area of $\triangle ABC$.

   (c) Find an equation of the plane containing $\triangle ABC$.

5. (8 points) Consider the planes described by the following equations.

$$2x - 3y - z = -5 \quad \text{and} \quad 6x - 9y - 3z = 18$$

(a) Are the planes parallel? Explain.

(b) Find the distance between the planes.

6. (6 points) Find the angle between the vectors $\vec{v} = -2\hat{\imath} + \hat{\jmath} + 6\hat{k}$ and $\vec{w} = 4\hat{\imath} + 3\hat{k}$.

7. (4 points) Find the domain of the vector-valued function
$\vec{f}(t) = \sqrt{t+2}\,\hat{\imath} + (5\cos t)\,\hat{\jmath} + \dfrac{1}{\sin t}\,\hat{k}$.

8. (6 points) Suppose $\dfrac{d\vec{r}}{dt} = (\sec^2 t)\hat{\imath} + (\sin t)\hat{\jmath} + 5e^{-2t}\hat{k}$. Find $\vec{r}(t)$ if $\vec{r}(0) = 4\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$.

9. (10 points) Consider the surface described by the equation $2z^2 = x^2 + y^2 - 4$.

   (a) Identify the surface.

   (b) Describe (or sketch) in detail the level curve obtained by fixing $z = 0$.

   (c) Describe (or sketch) in detail the level curve(s) obtained by fixing $y = 2$.

   (d) Sketch a rough graph of the surface.

10. (12 points) A projectile is launched from a bench above the ground. It is launched with an initial speed of 100 feet per second and at an angle of 30° above the horizontal. The projectile hits the ground after it has covered a horizontal distance of 276 ft. How high is the bench?

11. (8 points) A particle is moving along the helix described by the vector-valued function $\vec{r}(t) = (3\sin t)\hat{\imath} + (3\cos t)\hat{\jmath} + 5t\hat{k}$. Find the particle's unit tangent vector.

12. (4 points) Sketch or describe the surface in space defined by the equation $y = 16 - x^4$.

13. (4 points) Determine a vector-valued function whose graph is the plane curve described by the equation $xy + x^2 = 3$.

14. (8 points) Sketch the graph of the vector-valued function $\vec{r}(t) = -t^3\hat{\imath} + t\hat{\jmath}$. Draw arrows on your graph to indicate the curve's orientation. Then, without actually computing it, sketch the unit tangent vector at $t = 1$.

15. (4 points extra credit) For nonzero vectors $\vec{u}$ and $\vec{v}$, prove that the vector $\vec{w} = \|\vec{u}\|\vec{v} + \|\vec{v}\|\vec{u}$ bisects the angle between $\vec{u}$ and $\vec{v}$. (You may assume basic geometry facts about parallelograms.)