Math 173 - Test 2

March 21, 2018

Name ______________________

Score __________

Show all work. Supply explanations when necessary.

1. (10 points) Let $g(x, y) = \ln(x^2 - y)$.

   (a) What is the domain of g?

   (b) Evaluate $g(e, 0)$.

   (c) Sketch the level curve $g(x, y) = 0$.

   (d) Sketch the level curve $g(x, y) = 1$.

   (e) $z = g(x, y)$ explicitly defines z as a function of x and y and implicitly defines y as a function of x and z. Find a formula that **explicitly** defines y as a function of x and z.

2. (10 points) Consider the function f defined by

$$f(x,y) = \begin{cases} \dfrac{3xy^2}{x^2+y^4}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

(a) Determine $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ or show that it does not exist.

(b) Discuss the continuity of f.

3. (10 points) Let $h(x,y) = e^{x^2+xy+y^2}$.

(a) Find all values of x and y for which $h_x(x,y) = 0$ and $h_y(x,y) = 0$ simultaneously.

(b) If you were asked to compute $h_{xy}(x,y)$ and $h_{yx}(x,y)$, would you expect them to be equal? Briefly explain.

4. (10 points) The volume of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.

   (a) Assume that r and h are functions of the variable t. Use the chain rule to write a formula for dV/dt.

   (b) Now suppose that $r = e^t$ and $h = e^{-2t}$, for $t \geq 0$. Use your result from (a) to find dV/dt.

   (c) As t increases, does the volume of the cylinder in part (b) increase or decrease? Briefly explain how you know.

5. (10 points) Let $w = x^2 y z^2 + \sin(yz)$.

   (a) Determine the total differential dw.

   (b) Suppose (x, y, z) changes from $(2, 0, 1)$ to $(1.9, -0.2, 1.2)$. Use differentials to approximate Δw.

6. (10 points) Let $f(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$.

   (a) What is the domain of f?

   (b) Evaluate $f(1, 2, 3)$.

   (c) Discuss the continuity of f.

   (d) Describe the level surface $f(x, y, z) = 1$.

7. (12 points) Evaluate each limit or show that it does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x-y)^2}{x^2 + xy + y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,1)} \frac{y \sin x}{x(y+1)}$

8. (10 points) Assume that the following equation implicitly defines z as a function of x and y. Find the first partial derivatives of z.

$$x \ln y + y^2 z + z^2 = 8$$

9. (8 points) Consider the function $f(x, y, z) = x^2 y + 2xz^2 - 3y^2 z$. Choose two 3rd-order partial derivatives that you expect to be equal. Then compute both of them.

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10. (5 points) Use the definition of differentiability to show that $g(x, y) = x^3 - 2xy + 7y$ is differentiable on $\mathbb{R}^2$.

11. (5 points) The period T of a pendulum of length L is given by $T = 2\pi\sqrt{L}/\sqrt{g}$, where g is the acceleration due to gravity. A pendulum is moved from the Canal Zone, where $g = 32.09\,\text{ft/s}^2$, to Greenland, where $g = 32.23\,\text{ft/s}^2$. Because of the change in temperature, the length of the pendulum shrank from $2.5\,\text{ft}$ to $2.48\,\text{ft}$. Use differentials to approximate the change in the period of the pendulum.