

# Math 173 - Extra Credit

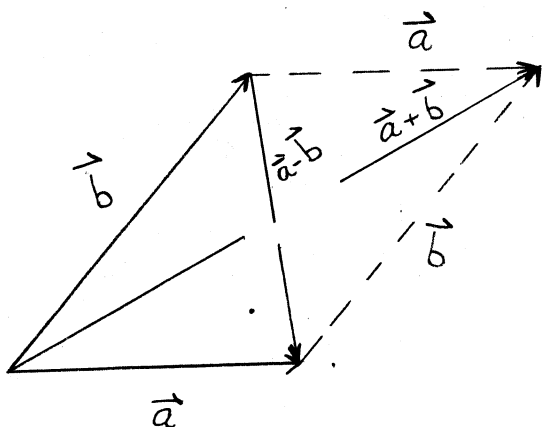
January 23, 2019

Name key  
Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. This problem is worth 1 extra credit point. It is due Monday, January 28, 2019.

Suppose  $\vec{a}$  and  $\vec{b}$  are vectors with  $\|\vec{a} - \vec{b}\| = \|\vec{a} + \vec{b}\|$ . What can you conclude about  $\vec{a}$  and  $\vec{b}$ ? Justify your answer. (Hint: I think it helps to sketch  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{a} - \vec{b}$ , and  $\vec{a} + \vec{b}$ .)

Approach #1 ...



$$\text{If } \|\vec{a} + \vec{b}\| = \|\vec{a} - \vec{b}\|,$$

THEN THE DIAGONALS OF A PARALLELOGRAM ARE CONGRUENT.

THEREFORE THE PARALLELOGRAM IS A RECTANGLE, AND IT FOLLOWS

THAT VECTORS  $\vec{a}$  &  $\vec{b}$  ARE

PERPENDICULAR.

$$\|\vec{a} - \vec{b}\| = \|\vec{a} + \vec{b}\| \Rightarrow \vec{a} \text{ \& \ } \vec{b} \text{ ARE ORTHOGONAL.}$$

Approach #2 ...

$$\|\vec{a} - \vec{b}\| = \|\vec{a} + \vec{b}\|$$

$$\Leftrightarrow (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$$

$$\Leftrightarrow \vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = \vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}$$

$$\Leftrightarrow -\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{b}$$

$$\Leftrightarrow \vec{a} \cdot \vec{b} = 0$$