

Math 173 - Quiz 1

January 17, 2019

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Think about the vector \vec{u} from the point $P = (-3, 2)$ to the point $Q = (-4, -5)$.

- (a) Find the component form of \vec{u} .

$$\vec{u} = \langle -4 - (-3), -5 - 2 \rangle = \langle -1, -7 \rangle$$
$$= -\hat{i} - 7\hat{j}$$

- (b) What angle does \vec{u} make with the positive x -axis?

\vec{u} IS IN THE 3RD QUADRANT
AND $\tan \theta = \frac{-7}{-1} = 7$

$$\tan^{-1}(7) \approx 81.87^\circ$$
$$\theta \approx 81.87^\circ + 180^\circ$$
$$= \boxed{261.87^\circ}$$

- (c) Compute $\|\vec{u}\|$.

$$\|\vec{u}\| = \sqrt{(-1)^2 + (-7)^2} = \sqrt{50} = 5\sqrt{2} \approx 7.07107$$

- (d) Determine a vector of magnitude 5 whose direction is opposite that of \vec{u} .

$$\|\vec{u}\| = \sqrt{50} = 5\sqrt{2}$$

$$-\frac{5}{5\sqrt{2}} \vec{u} = -\frac{1}{\sqrt{2}} \vec{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}} \right\rangle$$
$$= \frac{1}{\sqrt{2}} \hat{i} + \frac{7}{\sqrt{2}} \hat{j}$$

Turn over.

2. (2 points) Let $\vec{u} = 2\hat{i} - 3\hat{j}$ and let \vec{v} be the 2D vector of magnitude 4 that makes a 150° angle with the positive x -axis. Compute $2\vec{u} + \vec{v}$ and then find its magnitude.

$$\vec{u} = 2\hat{i} - 3\hat{j}$$

$$\begin{aligned}\vec{v} &= 4 \cos 150^\circ \hat{i} + 4 \sin 150^\circ \hat{j} \\ &= -2\sqrt{3}\hat{i} + 2\hat{j}\end{aligned}$$

$$2\vec{u} + \vec{v} = (4 - 2\sqrt{3})\hat{i} - 4\hat{j}$$

$$\|2\vec{u} + \vec{v}\| = \sqrt{(4 - 2\sqrt{3})^2 + (-4)^2} = \sqrt{44 - 16\sqrt{3}} \approx 4.03574$$

3. (2 points) Find a unit vector that is parallel to the graph of $y = \sqrt{x^2 + 1}$ at the point where $x = 2$. (Parallel to the graph means parallel to the tangent line.)

$$\frac{dy}{dx} = \frac{1}{2}(x^2 + 1)^{-1/2}(2x) = \frac{x}{\sqrt{x^2 + 1}}$$

$$\left. \frac{dy}{dx} \right|_{x=2} = \frac{2}{\sqrt{5}}$$

$$m = \frac{2}{\sqrt{5}} \Rightarrow \sqrt{5}\hat{i} + 2\hat{j}$$

$$\|\sqrt{5}\hat{i} + 2\hat{j}\| = \sqrt{5 + 4} = 3$$

Ans:

$$\frac{\sqrt{5}}{3}\hat{i} + \frac{2}{3}\hat{j}$$

- or -

$$-\frac{\sqrt{5}}{3}\hat{i} - \frac{2}{3}\hat{j}$$

4. (2 points) Determine a vector of magnitude 2 that is parallel to $\vec{w} = \hat{i} + 3\hat{j} - 5\hat{k}$.

$$\|\vec{w}\| = \sqrt{(1)^2 + (3)^2 + (-5)^2} = \sqrt{35}$$

$$\frac{2}{\sqrt{35}} \vec{w} = \frac{2}{\sqrt{35}} \hat{i} + \frac{6}{\sqrt{35}} \hat{j} - \frac{10}{\sqrt{35}} \hat{k}$$