Math 173 - Quiz 2

January 24, 2019

Name Key

Score ________

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Determine a vector of magnitude 5 that is parallel to $\vec{a} = -2\hat{\imath} + \hat{\jmath} + 4\hat{k}$.

$$\|\vec{u}\| = \sqrt{(-2)^2 + (1)^2 + (4)^2} = \sqrt{21}$$

$$\frac{5}{\sqrt{21}}\vec{u} = \frac{-10}{\sqrt{21}}\hat{\imath} + \frac{5}{\sqrt{21}}\hat{\jmath} + \frac{20}{\sqrt{21}}\hat{k}$$

2. (2 points) Determine the angle between the vectors $\vec{w} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ and $\vec{v} = 2\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$. Write your answer in degrees, rounded to the nearest hundredth.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \frac{2-12-20}{\sqrt{21}\,\sqrt{65}} = \frac{-30}{\sqrt{1365}}$$

$$\cos^{-1}\left(-\frac{30}{\sqrt{1365}}\right) \approx 2.51836$$

$$\approx 144.29^\circ$$

3. (1 point) Find the values of x for which $\vec{a} = x\hat{\imath} + \hat{\jmath} - 3\hat{k}$ and $\vec{b} = x\hat{\imath} + 2\hat{\jmath} + x\hat{k}$ are orthogonal.

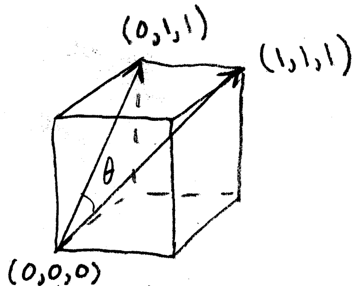
$$\vec{a}\cdot\vec{b} = 0 \Rightarrow x^2 + 2 - 3x = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x = 2 \text{ or } x = 1$$

4. (1 point) Determine the angle between the diagonal of a cube and the diagonal of one of its faces.



$\vec{u} = \hat{j} + \hat{k}$

$\vec{v} = \hat{i} + \hat{j} + \hat{k}$

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \dfrac{2}{\sqrt{2}\sqrt{3}} = \dfrac{2}{\sqrt{6}}$

$\theta = \cos^{-1}\left(\dfrac{2}{\sqrt{6}}\right) \approx 35.2644°$

5. (1 point) Find a unit vector that is orthogonal to both $\vec{u} = \hat{\imath}+\hat{\jmath}-3\hat{k}$ and $\vec{v} = 2\hat{\imath}-\hat{\jmath}+5\hat{k}$.

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -3 \\ 2 & -1 & 5 \end{vmatrix}$$

$= 2\hat{i} - 11\hat{j} + (-3)\hat{k}$

$\|\vec{u}\times\vec{v}\| = \sqrt{4+121+9} = \sqrt{134}$

Ans: $\dfrac{1}{\sqrt{134}}\left(2\hat{i} - 11\hat{j} - 3\hat{k}\right)$

6. (1 point) Find the projection of $\vec{b}$ onto $\vec{a}$ when $\vec{a} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\vec{b} = 4\hat{\imath} + 2\hat{\jmath} + \hat{k}$.

$\text{proj}_{\vec{a}}\,\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\,\vec{a} = \dfrac{12-2-2}{9+1+4}\,\vec{a} = \dfrac{8}{14}\,\vec{a} = \dfrac{4}{7}\,\vec{a} =$

$\dfrac{12}{7}\hat{i} - \dfrac{4}{7}\hat{j} - \dfrac{8}{7}\hat{k}$

7. (1 point) Give an example of three nonzero vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ in 3-space such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ but $\vec{b} \neq \vec{c}$.

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = \hat{i} + \hat{j} - \hat{k}$

$\vec{c} = 3\hat{i} - \hat{k}$

$\vec{a}\cdot\vec{c} = 3 - 3 = 0$

$\vec{a}\cdot\vec{b} = 1 + 2 - 3 = 0$

8. (1 point) Find the area of the triangle determined by the vectors $\vec{u} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{v} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$.

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 2 & -5 \end{vmatrix}$$

$= -9\hat{i} + 9\hat{j} + 0\hat{k}$

$= -9\hat{i} + 9\hat{j}$

Area $= \dfrac{1}{2}\|\vec{u}\times\vec{v}\|$

$= \dfrac{1}{2}\sqrt{162} = \boxed{\dfrac{9}{2}\sqrt{2}}$