Math 173 - Quiz 3
February 7, 2019

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (1 point) Determine parametric equations for the line passing through $(7, -3, 5)$ and parallel to the line with symmetric equations

$$\frac{x-2}{3} = y = \frac{2(z+1)}{5}.$$

$\vec{v} = 3\hat{i} + \hat{j} + \frac{5}{2}\hat{k}$

Point $(7, -3, 5)$ $\Rightarrow$

$x = 3t + 7$

$y = t - 3$

$z = \frac{5}{2}t + 5$

2. (2 points) Determine an equation of the plane passing through the points $P(1, 2, 3)$, $Q(-5, 3, -4)$, and $R(-3, -3, 3)$.

$\vec{PQ} = -6\hat{i} + \hat{j} - 7\hat{k}$

$\vec{PR} = -4\hat{i} - 5\hat{j}$

$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 1 & -7 \\ -4 & -5 & 0 \end{vmatrix} = \hat{i}(-35) - \hat{j}(-28) + \hat{k}(34)$

$\vec{n} = -35\hat{i} + 28\hat{j} + 34\hat{k}$

Plane: $-35x + 28y + 34z = -35 + 56 + 102$

$-35x + 28y + 34z = 123$

3. (2 points) Determine the angle between the planes given by

$$2x - 5y + 6z = 9 \quad \text{and} \quad 3x + y - 4z = 12.$$

Write your answer in degrees rounded to the nearest hundredth.

$\vec{n}_1 = 2\hat{i} - 5\hat{j} + 6\hat{k}$

$\vec{n}_2 = 3\hat{i} + \hat{j} - 4\hat{k}$

$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \, \|\vec{n}_2\|} = \frac{6 - 5 - 24}{\sqrt{65}\sqrt{26}} = \frac{-23}{\sqrt{1690}}$

$\theta \approx 124.02°$

Angle between planes is about

$180° - 124.02° = 55.98°$

TAKE-HOME PORTION OF QUIZ 3. DUE MONDAY.

4. (2 points) Find parametric equations for the line of intersection of the planes given by

$$x - y + 5z = -13 \quad \text{and} \quad 3x + 2y - z = 9.$$

$\vec{n}_1 = \hat{i} - \hat{j} + 5\hat{k}$

$\vec{n}_2 = 3\hat{i} + 2\hat{j} - \hat{k}$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 5 \\ 3 & 2 & -1 \end{vmatrix} = \hat{i}(-9) - \hat{j}(-16) + \hat{k}(5)$$

$$= -9\hat{i} + 16\hat{j} + 5\hat{k}$$

POINT? LET $z = 0$.

$x - y = -13$

$3x + 2y = 9$

$5x = -17$

$5y = 48$

$\left(-\frac{17}{5}, \frac{48}{5}, 0\right)$

PARAMETRIC EQN'S:

$x = -9t - \frac{17}{5}$

$y = 16t + \frac{48}{5}$

$z = 5t.$

5. (2 points) Determine the distance between the parallel planes

$$\underbrace{2x + 2x + 8z = -4}_{P_1} \quad \text{and} \quad \underbrace{2x + 2y + 8z = 15.}_{P_2}$$

POINT FROM P_1.

$(-1, -1, 0)$

P_2:

$\vec{n} = 2\hat{i} + 2\hat{j} + 8\hat{k}$

$$\text{DISTANCE} = \frac{|2(-1) + 2(-1) + 8(0) - 15|}{\sqrt{4 + 4 + 64}} = \frac{19}{\sqrt{72}} \approx 2.2392$$

6. (1 point) Compute the distance from the point $(4, -3, 5)$ to the line described by

$$\frac{x-2}{5} = \frac{y+6}{2} = \frac{z+1}{3}.$$

$P\ (2, -6, -1)$

$Q\ (4, -3, 5)$

$\vec{v} = 5\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{PQ} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$$\vec{PQ} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 5 & 2 & 3 \end{vmatrix}$$

$$= \hat{i}(-3) - \hat{j}(-24) + \hat{k}(-11)$$

$$= -3\hat{i} + 24\hat{j} - 11\hat{k}$$

$$\text{DISTANCE} = \frac{\|\vec{PQ} \times \vec{v}\|}{\|\vec{v}\|} = \sqrt{\frac{9 + 576 + 121}{25 + 4 + 9}} = \sqrt{\frac{706}{38}} \approx 4.3103$$