Math 173 - Quiz 7

March 28, 2019

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Suppose $w = x^2 - y^2$, $x = s\cos t$, and $y = s\sin t$. Use the chain rule to find $\partial w/\partial t$ when $s = 3$ and $t = \pi/4$.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} = (2x)(-s\sin t) + (-2y)(s\cos t)$$

When $s = 3$ & $t = \frac{\pi}{4}$ ...

$x = \frac{3\sqrt{2}}{2}$, $y = \frac{3\sqrt{2}}{2}$

$$= (3\sqrt{2})\left(-\frac{3\sqrt{2}}{2}\right) + (-3\sqrt{2})\left(\frac{3\sqrt{2}}{2}\right)$$

$$= -18$$

2. (3 points) Suppose z is implicitly defined as a function of x and y by the equation

$$x\ln y + y^2 z = 8 - z^2.$$

$F(x,y,z) = x\ln y + y^2 z + z^2 = 8$

Find $\partial z/\partial y$.

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z} = \frac{-\left(\frac{x}{y} + 2yz\right)}{y^2 + 2z} = \frac{-x - 2y^2 z}{y^3 + 2yz}$$

3. (4 points) Show that $f(x,y) = x^2 - y$ is differentiable by using the definition of differentiability.

$z = f(x,y)$

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y) = \left[(x+\Delta x)^2 - (y+\Delta y)\right] - \left[x^2 - y\right]$$

$$= 2x\,\Delta x + \Delta x^2 - \Delta y$$

$$= 2x\,\Delta x - \Delta y + \Delta x\,\Delta x + 0\,\Delta y$$

$$= \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

where $\epsilon_1 = \Delta x$ and $\epsilon_2 = 0$

Since this holds for all (x,y) and $(\epsilon_1, \epsilon_2) \to (0,0)$ as $(\Delta x, \Delta y) \to (0,0)$, f is differentiable everywhere on $\mathbb{R}^2$.