Math 173 - Quiz 9
April 11, 2019

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Find and classify the critical points of $f(x,y) = x^3 - 3xy + y^3 + 3$. Identify all relative extreme values.

$f_x(x,y) = 3x^2 - 3y = 0 \Rightarrow y = x^2$

$f_y(x,y) = -3x + 3y^2 = 0$

$-3x + 3x^4 = 0$

$3x(x^3 - 1) = 0$

$x = 0, \; x = 1$ (1 HAS ONLY ONE REAL CUBE ROOT)

$y = 0 \qquad y = 1$

CRIT PTS ARE $(0,0)$ AND $(1,1)$.

$$d(x,y) = \begin{vmatrix} 6x & -3 \\ -3 & 6y \end{vmatrix} = 36xy - 9$$

$d(0,0) = -9 < 0 \Rightarrow$ $\boxed{(0,0,3) \text{ IS A SADDLE POINT.}}$

$d(1,1) = 27 > 0$ AND $f_{xx}(1,1) = 6 > 0$

$\Rightarrow$ $\boxed{f(1,1) = 2 \text{ IS A RELATIVE MIN}}$

$f_x(x,y) = 4x+1$, $f_y(x,y) = -2y$, $g_x(x,y) = y - (\cos x)e^{\sin x} - 3$

$g_y(x,y) = x$

2. (5 points) Suppose we are given the two functions

$$f(x,y) = 2x^2 + x - y^2 + 4, \qquad g(x,y) = xy - e^{\sin x} - 3x$$

and we wish to solve the system of nonlinear equations $f(x,y) = 0$, $g(x,y) = 0$. We can approximate a solution using linearizations and Newton's method. (Throughout this problem, round to five digits beyond the decimal point.)

(a) Let $(x_0, y_0) = (2.25, 4)$ be our initial guess at the solution. Let $f_0(x,y)$ and $g_0(x,y)$ be the linearizations of f and g at the point (x_0, y_0). Find $f_0(x,y)$ and $g_0(x,y)$.

$f_0(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$= 0.375 + 10(x-2.25) - 8(y-4) = 10x - 8y + 9.875 = f_0(x,y)$

$g_0(x,y) = g(x_0,y_0) + g_x(x_0,y_0)(x-x_0) + g_y(x_0,y_0)(y-y_0)$

$= 0.07273 + 2.36771(x-2.25) + 2.25(y-4) = 2.36771x + 2.25y - 14.25462 = g_0(x,y)$

(b) Solve the linear system of equations

$$f_0(x,y) = 0, \qquad g_0(x,y) = 0.$$

$10x - 8y = -9.875$

$2.36771x + 2.25y = 14.25462$

CALCULATOR SAYS ...

$x = 2.21560$

$y = 4.00388$

(c) Let (x_1, y_1) be the solution of the linear system in part (b). It represents an improved guess at the solution of the original system. Compute $f(x_1, y_1)$ and $g(x_1, y_1)$.

$f(x_1,y_1) = 0.00231$

$g(x_1,y_1) = 0.00040$