

**Math 173 - Test 2**

March 21, 2019

Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations where necessary.

---

1. (4 points) Let  $\vec{r}(t) = 2 \sin 5t \hat{i} + 8\hat{j} - 2 \cos 5t \hat{k}$ . Compute  $\|\vec{r}'(t)\|$  and then briefly explain (without computing) why you would expect that  $\vec{r}(t) \cdot \vec{r}'(t) = 0$ .

2. (6 points) Determine  $\vec{r}(t)$  if  $\frac{d\vec{r}}{dt} = \frac{1}{1+t^2} \hat{i} + \frac{1}{t^2} \hat{j} + \frac{1}{t} \hat{k}$  and  $\vec{r}(1) = 2\hat{i}$ .

3. (4 points) A object is falling in such a way that its position at time  $t$  is given by

$$\vec{r}(t) = 171\sqrt{3}t \hat{i} + (-4.9t^2 + 171t + 2) \hat{j}.$$

What is the maximum height of the object?

4. (8 points) A projectile is fired from 5 feet above the ground with an initial speed of 2800 ft/s. The projectile is supposed to hit a target 1500 feet away and 5 feet above the ground. Find the initial angle. (Ignore all forces except gravity and use  $g = 32 \text{ ft/s}^2$ .)

5. (2 points) If an object is moving along a straight line, what can be said about the principal unit normal vector?

6. (6 points) For a smooth curve described by  $\vec{r}(t)$ , the following facts are known:

- $\hat{T}(t_0) = \frac{3}{5}\hat{i} - \frac{4}{5}\hat{k}$
- $\hat{T}'(t_0) = -4\hat{i} - 3\hat{k}$
- $\|\vec{r}'(t_0)\| = 10$

(a) Find  $\vec{r}'(t_0)$ .

(b) Find  $\hat{N}(t_0)$ .

7. (8 points) Determine  $\hat{N}(0)$  if  $\vec{r}(t) = \sqrt{2}t\hat{i} + e^t\hat{j} + e^{-t}\hat{k}$ .  
(Hint: It may be helpful to recognize that  $2 + e^{2t} + e^{-2t} = (e^t + e^{-t})^2$ .)
8. (5 points) An object spirals around the  $x$ -axis on the space curve described by  $\vec{r}(t) = 7t\hat{i} + 2\sin t\hat{j} + 2\cos t\hat{k}$ . Determine the length of the path from  $t = 0$  to  $t = \pi$ .
9. (9 points) Let  $z = f(x, y) = 2x^2 - xy$ .
- (a) Compute the total differential  $dz$ .
- (b) Use differentials to approximate  $\Delta z$  if  $(x, y)$  changes from  $(1, 1)$  to  $(0.98, 1.03)$ .
- (c) Referring to part (b), compute the exact value of  $\Delta z$ .

10. (5 points) Determine the curvature as a function of  $x$  for the plane curve described by  $y = \ln x$ .

11. (15 points) Determine each limit or explain why the limit does not exist.

(a) 
$$\lim_{(x,y) \rightarrow (2,1)} \frac{x - y - 1}{\sqrt{x - y} - 1}$$

(b) 
$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^2}$$

(c) 
$$\lim_{(x,y) \rightarrow (0,1)} \frac{\sin(x^2 + y - 1)}{3y + x^2 - 3}$$

12. (2 points) True or false: The curvature at any point on a circle is constant and equal to the radius of the circle.

13. (10 points) Let  $g(x, y, z) = 4x^2 - y^2 + 4z^2$ .

(a) What is the domain of  $g$ ?

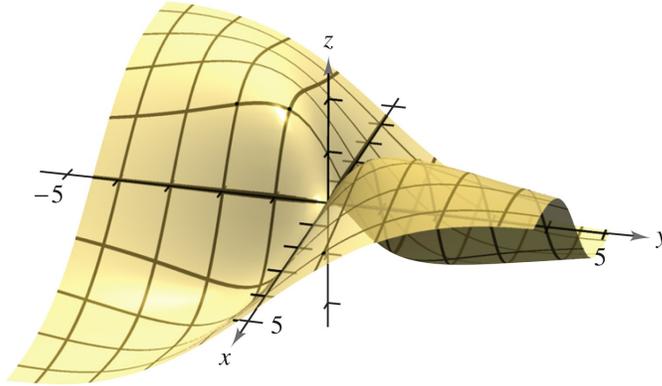
(b) Evaluate  $g(1, -3, 1)$ .

(c) Describe the level surface  $g(x, y, z) = 9$ .

(d) Describe the level surface  $g(x, y, z) = -9$ .

(e) At which points is  $g$  continuous?

14. (6 points) The graph of  $z = f(x, y)$  is shown below. Look at the point on the graph where  $x = 1$  and  $y = -3$ .



- (a) Based on the graph, is the number  $f_x(1, -3)$  positive or negative?
- (b) Based on the graph, is the number  $f_y(1, -3)$  positive or negative?
- (c) Which is greater:  $|f_x(1, -3)|$  or  $|f_y(1, -3)|$ ?
15. (10 points) Let  $h(x, y) = x \cos xy^2$ .
- (a) Compute  $h_x$ .
- (b) Compute  $h_y$ .
- (c) Would you expect that  $h_{xy} = h_{yx}$ ? Explain.
- (d) Compute  $h_{yx}$ .