Math 200 - Test 1

September 22, 2010

Name key

Score _____

Show all work to receive full credit (even on multiple-choice problems). Supply explanations when necessary.

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1. (4 points) Clearly state the 4 steps of the problem-solving process (in order).

   ① UNDERSTAND THE PROBLEM

   ② DEVISE A PLAN

   ③ CARRY OUT THE PLAN

   ④ LOOK BACK

2. (1 point) When using the 4-step, problem-solving process which one of these strategies WOULD BE considered part of understanding the problem?

   (a) Look for a pattern. ← DEVISE A PLAN

   (b) Keep an accurate record of your work. ← CARRY OUT PLAN

   (c) Construct a table or draw a picture. ← DEVISE A PLAN

   (d) Determine what information is unknown.

3. (3 points) State three different strategies for carrying out the plan.

   i) IMPLEMENT THE PLAN YOU DEVISED

   ii) KEEP A RECORD OF YOUR WORK

   iii) CHECK WORK AT EACH STEP

4. (1 point) When using the 4-step, problem-solving process which one of these strategies WOULD NOT BE considered part of devising a plan?

   (a) Look for a pattern.

   (b) Determine what is known and unknown. ← UNDERSTAND THE PROBLEM

   (c) Work backward.

   (d) Guess and check.

5. (1 point) Jerry defined a sequence as follows:

> The first term is the number 8. To get any subsequent term, multiply the previous term by 4 and then subtract 5.

What type of sequence did Jerry define?

(a) An arithmetic sequence

(b) A geometric sequence

(c) A sequence with a fixed higher-order difference

(d) A recursive sequence

↑ JERRY OBTAINS EACH TERM FROM PREVIOUS TERMS.

6. (1 point) Compute the following sum:

$$1+2+3+4+\cdots+136+137$$

$$137+136+\text{---}+1$$

137 PAIRS OF 138

$$\text{Sum} = \frac{(137)(138)}{2} = 9453$$

(a) 9453

(b) 18,906

(c) 9316

(d) 9248

7. (1 point) Which one of the following is NOT true about the empty set?

(a) Its cardinality is zero.

(b) It is a subset of every set.

(c) It can be denoted by $\{\emptyset\}$.

(d) It is equivalent to itself.

8. (1 point) Let $A = \{1, 2\}$. How many subsets does A have?

(a) 4 $\quad \emptyset, \{1\}, \{2\}, \{1,2\}$

(b) 3

(c) 2

(d) 1

9. (1 point) Which one of the following sets is well defined?

(a) $\{x \mid x \text{ is a big number}\}$

(b) The set of all nice days

(c) $\{1, 7, f, 9, q, \frac{2}{3}\}$ ← WE KNOW EXACTLY WHAT'S IN IT.

(d) $\{p \mid p \text{ is a pretty flower}\}$

10. (3 points) Rewrite each of the following statements using mathematical symbols.

(a) The empty set is a subset of the set A.

$\phi \subseteq A$ OR $\{\} \subseteq A$

(b) 6 is an element of the set H

$6 \in H$

(c) The sets P and Q are equivalent.

$P \sim Q$

11. (3 points) The following sequence could be either arithmetic or geometric depending on the numbers replacing the blanks. Find the first five terms of both possible sequences.

2, ____, 18, ____, ____, ...

ARITHMETIC:

DIFFERENCE MUST BE $\frac{18-2}{2} = \frac{16}{2} = 8$

2, 10, 18, 26, 34, ...

GEOMETRIC:

$18 = 2 \cdot r^2 \Rightarrow r^2 = 9$

$\Rightarrow r = \pm 3$

2, 6, 18, 54, 162, ...

OR 2, -6, 18, -54, 162, ...

12. (2 points) Kevin made the following conjecture:

> If a fixed number is added to each term of a geometric sequence, the new sequence will also be a geometric sequence.

Give a counterexample to disprove the conjecture.

START WITH THE GEO. SEQUENCE 1, 2, 4, 8, 16, ...

ADD ONE TO EACH TERM: 2, 3, 5, 9, 17, ...

NOT GEOMETRIC SINCE $\frac{3}{2} \neq \frac{5}{3}$

13. (1 point) Who is given credit for devising the technique we use for finding the sum of the terms of an arithmetic sequence?

(a) Leonhard Euler

(b) Kurt Gödel

(c) Gerolamo Cardano

(d) Carl Gauss

14. (4 points) Problem-solving problem: In our class, there are six times as many women as men. Using variables, write a formula relating the number of women to the number of men. (It should be clear how you have used the problem-solving process.)

Let w = # of women in class

Let m = # of men in class

There are 6 x as many women means 6 women for every man

Ratio of women to men is $\frac{w}{m} = \frac{6}{1}$

This means $\boxed{w = 6m}$

Check: If there are 4 men there must be 24 women. $24 = 6(4)$ Yes!

15. (2 points) Sally made the following conjecture:

The sum of two even numbers is an even number.

In order to disprove the conjecture, Jonathon gave the counterexample $3 + 5 = 8$. What is wrong with Jonathon's reasoning?

Jonathon gave neither an example nor a counterexample. He found the sum of two odd numbers. This conjecture is about the sum of two even numbers.

16. (3 points) Which one of these numbers is the 691st term of the following arithmetic sequence?

$$18, 25, 32, 39, 46, \ldots$$
$$\overset{+7}{\vee} \quad \overset{+7}{\vee} \quad \overset{+7}{\vee} \quad \overset{+7}{\vee}$$

N^{TH} term is $7N + 11$

(a) 4914

(b) 4848

(c) 4826

(d) 7594

691^{ST} term $= 7(691) + 11$

$= 4848$

17. (2 points) A sequence is defined recursively as follows:

$$B_1 = 5; \quad B_n = 2 \cdot B_{n-1} - 3, \text{ for } n = 2, 3, 4, \ldots$$

Find B_3 (the third term of the sequence).

(a) 11

(b) 7

(c) 5

(d) 27

$B_2 = 2B_1 - 3 = 2(5) - 3 = 7$

$B_3 = 2B_2 - 3 = 2(7) - 3 = 11$

18. (6 points) Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5\}$, and $C = \{4, 5, 6\}$. In each of the following, insert an appropriate symbol $\sim$, $\in$, or $\subseteq$ to make the statement true.

(a) $2 \_\_\in\_\_ A$

(b) $B \_\_\subseteq\_\_ A$

(c) $C \_\_\sim\_\_ B$

(d) $\emptyset \_\_\subseteq\_\_ C$

(e) $A \_\_\sim\_\_ A$ OR $A \subseteq A$

(f) $\{5\} \_\_\subseteq\_\_ C$

19. (1 point) For which one of these sets is it true that $n(A) = 1$?

(a) $A = \emptyset$ ← ZERO ELEMENTS

(b) $A = \{0, 1\}$ ← TWO ELEMENTS

(c) $A = \{\emptyset\}$ ← ONE ELEMENT

(d) $A = \{1, 11, 111, 1111, \ldots\}$ ← INF MANY ELEMENTS

20. (2 points) There are 511 terms in the following sequence. Find the sum of the terms.

$$1, 4, 7, 10, \ldots, 1528, 1531$$

(a) 261,632

(b) 391,170.5

(c) 1,172,746

(d) 391,426

$1 + 4 + \cdots + 1528 + 1531$

$1531 + 1528 + \cdots + 4 + 1$

511 PAIRS OF 1532

Sum is $\frac{(511)(1532)}{2} = 391,426$

21. (3 points) Consider the following sequence:

$$1, 3, 8, 16, 27, 41, \ldots$$

1, 3, 8, 16, 27, 41
2 5 8 11 14
3 3 3 3

FIXED 2ND DIFF.

(a) Is this sequence arithmetic? How do you know?

No! $1+2=3$ BUT $3+2 \neq 8$ THE TERMS DON'T HAVE A CONSTANT DIFFERENCE.

(b) Is this sequence geometric? How do you know?

No! $\frac{3}{1} \neq \frac{8}{3}$ THE TERMS DON'T HAVE A CONSTANT RATIO.

(c) Find 3 terms that continue a possible pattern.

1 3 8 16 27 41 58 78 101
2 5 8 11 14 17 20 23

22. (2 points) What does it mean for two sets to be equivalent?

TWO SETS ARE EQUIVALENT IF THERE IS A ONE-TO-ONE CORRESPONDENCE FROM ONE TO THE OTHER.

23. (1 point) Suppose U is the set of all PSC math students and M is the set of all Math 200 students. Describe an element of $\overline{M}$.

AN ELEMENT OF $\overline{M}$ IS A PSC MATH STUDENT WHO IS NOT IN MATH 200.

24. (1 point) Let W be the set of all whole numbers greater than 101. Write W using set-builder notation.

$W = \{ x \mid x \text{ IS A WHOLE NUMBER AND } x > 101 \}$