Math 200 - Test 2

October 20, 2010

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (1 point) What number comes right after 6566_{seven}?

   (a) 6567_{seven}
   (b) 6570_{seven}
   (c) 6700_{seven}
   (d) 6600_{seven}

2. (1 point) Choose the addition model that best fits the following problem situation: *William has toy train cars of assorted shapes and sizes. He connects three cars to form a train that is 9 in long. Then he connects two cars to form a train that is 7 in long. If he connects both trains, what will be the length of the new train?*

   (a) number line model
   (b) set partition model
   (c) combination model
   (d) set model

3. (2 points) Use any algorithm to compute the following sum.

   $$333_{\text{four}} + 123_{\text{four}} + 321_{\text{four}} + 323_{\text{four}}$$

   (a) 1100_{four}
   (b) 3032_{four}
   (c) 10203_{four}
   (d) 2332_{four}

   $$\begin{array}{r} {\scriptstyle 2\ \ 2\ \ \ } \\ 3_1\ 3_1\ 3 \\ 1\ \ 2\ 3_2 \\ 3_1\ 2_1\ 1 \\ +\ 3_0\ \ 2\ 3_2 \\ \hline 3\ 0\ 3\ 2 \end{array}$$

4. (1 point) Suppose A is the set of all PSC students and B is the set of all females. Which one of the following is a description of an element of $\overline{B} - A$?

   (a) a male who is not a PSC student
   (b) a female who is not a PSC student
   (c) a female PSC student
   (d) a male PSC student

   NOT A FEMALE, NOT A PSC STUDENT

5. (1 point) Choose the multiplication model that best fits the following problem situation: *Mrs. Chesney has 8 packages of 6 markers. How many markers does she have in all?*

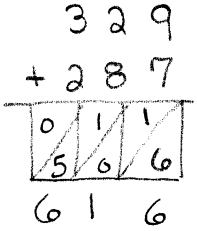
   (a) Cartesian product model
   (b) set partition model
   (c) area/array model
   (d) repeated addition model

6. (2 points) Use any of the algorithms we discussed in class, except the standard algorithm, to compute $329 + 287$.

INTRODUCTORY / PARTIAL SUMS

$$\begin{array}{r} 329 \\ +\ 287 \\ \hline 16 \\ 100 \\ 500 \\ \hline 616 \end{array}$$

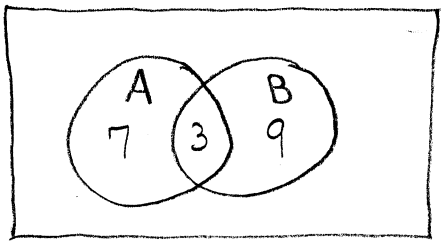
LATTICE



SCRATCH

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 329 \\ +\ 287 \\ \hline 616 \end{array}$$

7. (2 points) Use a two-set Venn diagram to help you determine $n(A \cup B)$ if $n(A \cap B) = 3$, $n(A) = 10$, and $n(B) = 12$.



$n(A \cup B) = 19$

8. (2 points) Convert 473_{eight} to base ten.

$4 \times 8^2 + 7 \times 8^1 + 3 \times 8^0$

$= 4(64) + 7(8) + 3 = 256 + 56 + 3$

$= 315$

9. (3 points) Rewrite each expression using the indicated property, and only that property, exactly one time.

(a) *Commutative property of addition:* $8 \cdot (3 + 1) = 8 \cdot (1 + 3)$

(b) *Commutative property of multiplication:* $2 \cdot (5x + 7)$

$= (5x + 7) \cdot 2$ or $2[x(5) + 7]$

(c) *Associative property of addition:* $2y + (2 + 8x) + 4$

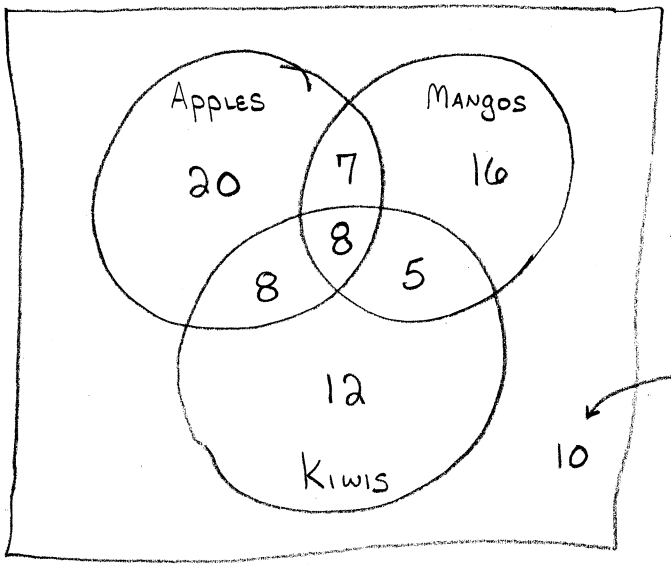
$= (2y + 2) + 8x + 4$

or $2y + 2 + (8x + 4)$

10. (5 points) Mr. Strand asked 86 students at his school about the kinds of fruit they normally eat. Here is what he found:

- 43 eat apples
- 36 eat mangos
- 33 eat kiwis
- 13 eat kiwis and mangos
- 15 eat apples and mangos
- 16 eat kiwis and apples
- 8 eat all three

(a) Use a three-set Venn diagram to organize this information.



$20+7+16+8$
$+8+5+12 = 76$

86
$-\ 76$
10

(b) How many students do not eat any of these fruits?

10 students

(c) How many students eat two of the fruits, but not all three?

$7+8+5 =$ 20 students

(d) How many students eat only one kind of fruit?

$20+16+12 =$ 48 students

11. (3 points) The following number chart can be used to compute sums and differences.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	(19)
20	21	22	23	24	25	26	27	28	[29]
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

(a) To compute $52 - 24$, one could start at 52, then count straight up two rows and back four columns. What subtraction model is being used?

I'D ACCEPT NUMBER LINE WITH A SUITABLE EXPLANATION.

THE TAKE AWAY MODEL -- TAKE AWAY 20 AND THEN TAKE AWAY 4.

(b) Explain how the missing addend model could be used to compute $32 - 19$.

$32 - 19 = \square \iff 19 + \square = 32$

START AT 19 AND COUNT ON TO 32... 1 ROW AND 3 COLUMNS $\Rightarrow 32 - 19 = 13$

12. (3 points) Convert 109 to base two.

$2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$

$2^6 = 64 \overline{)109}\ (1$ (64)

$2^5 = 32 \overline{)45}\ (1$ (32)

$2^4 = 16 \overline{)13}\ (0$ (0)

$2^3 = 8 \overline{)13}\ (1$ (8)

$2^2 = 4 \overline{)5}\ (1$ (4)

$2^1 = 2 \overline{)1}\ (0$ (0)

$2^0 = 1 \overline{)1}\ (1$

$109_{TEN} = 1101101_{TWO}$

13. (3 points) Use any **two** of the strategies for mastering basic addition facts to compute $9 + 7$. Show work or explain your reasoning.

DOUBLES:

$9 + 7 = 2 + 7 + 7$

$= 2 + 14 = 16$

MAKING 10:

$9 + 7 = 9 + 1 + 6$

$= 10 + 6 = 16$

14. (1 point) Choose the subtraction model that best fits the following problem situation: *There are 7 chairs in the first row and 4 chairs in the second row. How many more chairs are in the first row?*

(a) take-away model

(b) comparison model

(c) set partition model

(d) missing addend model

15. (1 point) What algorithm is being illustrated here?

$$\begin{array}{r} 72 \\ -\,38 \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 72 \;+\; 2 \\ -\,(38 \;+\; 2) \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 74 \\ -\,40 \\ \hline 34 \end{array}$$

(a) scratch subtraction algorithm

(b) fast subtraction algorithm

(c) equal-additions algorithm

(d) nice-numbers algorithm

16. (1 point) What is the place value of the digit 3 in the numeral 53115_{six}?

(a) 6^3

(b) $3 \cdot 6^3$

(c) 3

(d) 1000

17. (1 point) Which one of the following sets is closed under addition?

(a) $\{0, 1\}$

(b) $\{0, 2, 4, 6, 8, \ldots\}$ THE SUM OF TWO EVENS IS AN EVEN.

(c) $\{1, 3, 9, 27, 81, \ldots\}$

(d) $\{1, 3, 5, 7, 9, \ldots\}$

18. (1 point) Choose the multiplication model that best fits the following problem situation: *John has 7 shirts and 9 ties. How many shirt/tie combinations does he have?*

(a) comparison model

(b) repeated addition model

(c) area/array model

(d) Cartesian product model

19. (3 points) Use a multiplication model to illustrate the following fact.

$$3 \cdot (x + 4) = 3 \cdot x + 3 \cdot 4$$

REPEATED ADDITION

$3(x+4) = (x+4) + (x+4) + (x+4)$

$= x + x + x + 4 + 4 + 4$

$= 3x + 3(4)$

AREA

	x	4
3	3x	3(4)

$3(x+4) = 3x + 3(4)$

20. (3 points) Use any of the algorithms we discussed in class, except the standard algorithm, to compute $529 - 287$.

EQUAL ADDITIONS ALGORITHM

$$\begin{array}{r} 529 \\ -\ 287 \end{array} \quad +3 \rightarrow \quad \begin{array}{r} 532 \\ -\ 290 \end{array} \quad +10 \rightarrow \quad \begin{array}{r} 542 \\ -\ 300 \\ \hline 242 \end{array}$$

$529 - 287 = \boxed{242}$

21. (3 points) Use rectangular arrays to model and expand $(2x + 1)(x^2 + 3x + 2)$.

	x^2	$3x$	2
$2x$	$2x^3$	$6x^2$	$4x$
1	x^2	$3x$	2

$2x^3 + 6x^2 + 4x$

$+ \quad x^2 + 3x + 2$

$\boxed{2x^3 + 7x^2 + 7x + 2}$

22. (1 point) Explain why 300653_{six} cannot be a correctly written base-six numeral.

6 IS NOT A DIGIT IN BASE SIX.

23. (2 points) Let $A = \{1, 2, 3\}$ and $B = \{1, 5\}$.

(a) Determine $B \times A$.

$B \times A = \{(1,1), (1,2), (1,3), (5,1), (5,2), (5,3)\}$

(b) Determine $A - B$.

$A - B = \{2, 3\}$

24. (2 points) Use a three-set Venn diagram on the next page (one for each part) to shade the region corresponding to each of these sets. Label your diagrams.

(a) $A \cap \overline{B} \cap \overline{C}$

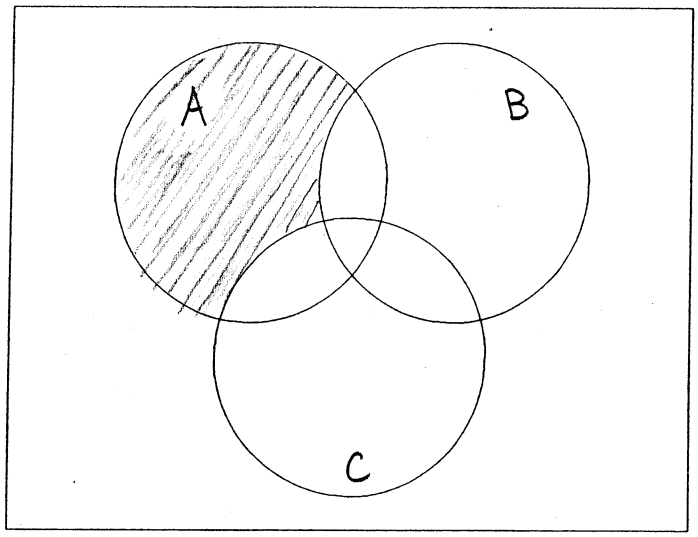
See next page.

(b) $(B \cap C) - A$

25. (2 points) Write the first ten counting numbers in base three.

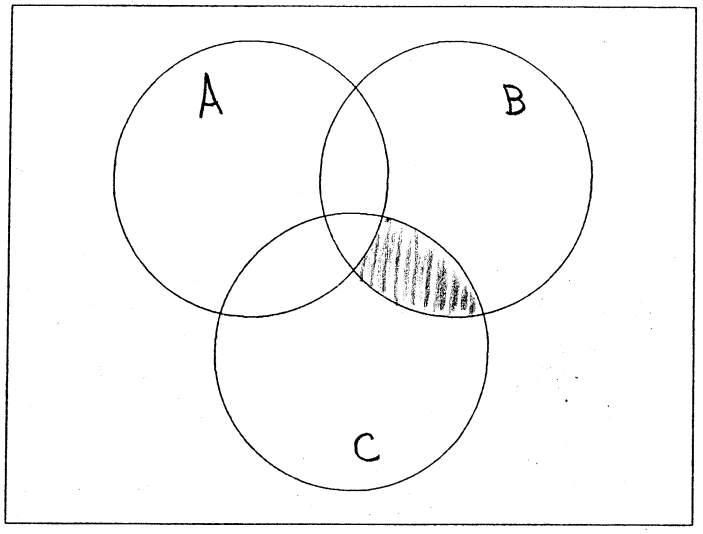
1, 2, 10, 11, 12, 20, 21, 22, 100, 101

(a)



$A \cap \overline{B} \cap \overline{C}$

(b)



$(B \cap C) - A$