Math 200 - Test 3

November 17, 2010

Name key

Score _____

Show all work. Supply explanations where necessary.

1. (1 point) Choose the division model that best fits the following problem situation: *Sabrina has 18 stickers for her sticker book. If she puts 3 stickers on each page, how many pages does she fill?*

"How many groups?"

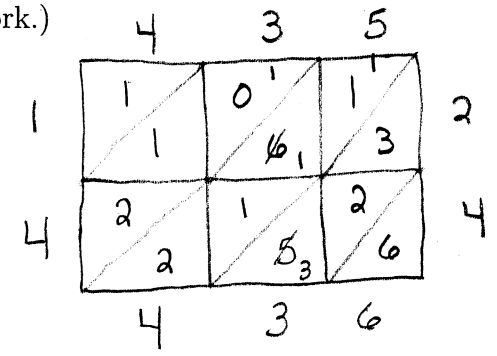
   (a) set partition model
   (b) repeated subtraction model
   (c) missing factor model
   (d) charged field model

2. (1 point) Which one of the following is a legitimate test for divisibility by 12?

   (a) An integer is divisible by 12 if and only if it is divisible by both 2 and 6.
   (b) An integer is divisible by 12 if and only if it is divisible by both 3 and 4.
   (c) An integer is divisible by 12 if and only if the sum of its digits is divisible by 12.
   (d) All of the above or none of the above

3. (3 points) Use any multiplication algorithm to compute $435_{\text{seven}} \times 24_{\text{seven}}$. (Use the back of the last page if you need more room to work.)

   (a) $10\,440_{\text{seven}}$
   (b) $14\,106_{\text{seven}}$
   (c) $14\,136_{\text{seven}}$
   (d) $14\,436_{\text{seven}}$



4. (1 point) Consider the following conjecture:

   If $x \mid 3y$, then $x \mid y$.

   Which one of the following is a counterexample?

   (a) $2 \mid (3 \cdot 6)$ and $2 \mid 6$.
   (b) $6 \mid (3 \cdot 2)$ and $6 \nmid 2$
   (c) $5 \nmid (3 \cdot 7)$ and $5 \mid 7$
   (d) The conjecture is true.

5. (1 point) Which one of these is an example of an integer x for which $|x| = -x$?

   (a) -1.352
   (b) 5
   (c) -117
   (d) None of the above

$|-117| = 117 = -(-117)$

6. (2 points) Give a brief but thorough explanation for why $0 \div 0$ is not defined.

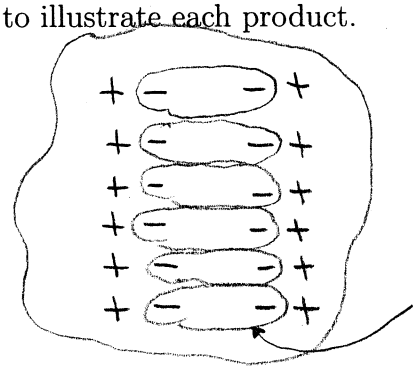
According to missing factor model, $0 \div 0 = k$ if and only if k is the unique number such that $k \cdot 0 = 0$.

This is true for any number k. There is no unique such number!

7. (4 points) Use a different model to illustrate each product.

(a) $-6 \times (-2)$

Charges: Start with zero. Take out 6 groups of 2 negs.



Remove these.

$-6 \times (-2) = 12$

(b) -4×5

Pattern:

$3 \times 5 = 15$
$2 \times 5 = 10$
$1 \times 5 = 5$
$0 \times 5 = 0$

Decrease factor by 1 $\Rightarrow$ decrease product by 5

$-1 \times 5 = -5$
$-2 \times 5 = -10$
$-3 \times 5 = -15$
$-4 \times 5 = -20$

(c) Explain why the repeated addition model is not a good model for illustrating either one of the products above.

It's hard to imagine what it means to repeatedly add something a negative number of times.

8. (4 points) Test the following number for divisibility by ~~2~~, ~~3~~, ~~4~~, ~~5~~, 6, ~~8~~, ~~9~~, ~~10~~, and ~~12~~.

41064276641149669988940

2: Yes, $2 \mid 0$

4: Yes, $4 \mid 40$

5: Yes, $5 \mid 0$

10: Yes, ends in 0

8: No, $8 \nmid 940$

Sum of digits is 114

Sum of digits is 6.

$\Downarrow$

3: Yes, $3 \mid 6$

9: No, $9 \nmid 6$

6: Yes, because yes for 2 & 3

12: Yes, because yes for 3 & 4

9. (1 point) Choose the division model that best fits the following problem situation: *April has 35 pieces of Halloween candy to divide evenly among 7 children. How much candy does each child get?*

"How much in each group?"

(a) charged field model

(b) repeated subtraction model

(c) missing factor model

(d) set partition model

10. (1 point) Which one of the following means the same as $a \mid b$?

(a) a divided by b

(b) a is a multiple of b

(c) a is a divisor of b

(d) There exists an integer k such that $a \div b = k$.

a divides b

11. (1 point) Suppose n and m are integers. Which one of the following is equal to $-n \times (-m)$?

(a) $n \times m$

(b) $-(n \times m)$

(c) $-n \times m$

(d) $n \times (-m)$

12. (1 point) Compute $-3 - (-3) - 3 + (-3) - 3 - (-3)$.

(a) 0

(b) -9

(c) -6

(d) 3

$\underbrace{-3+3}_{0} + \underbrace{(-3)+(-3)}_{-6} + \underbrace{(-3)+3}_{0}$

13. (1 point) Which one of the following is the additive inverse of the expression $-5x+2-y$?

(a) $5x + 2 - y$

(b) $5x - 2 + y$

(c) 1

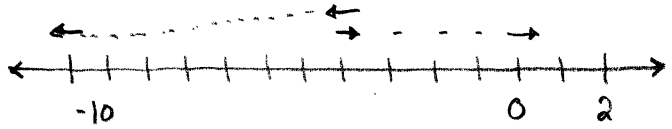
(d) $\dfrac{1}{5x - 2 + y}$

14. (3 points) Trevor used the number line to model $-4-7$. Here is what he said:

> "Start at zero facing right. Turn around, go four. Back-up seven. You end up at three. Therefore, $-4-7=3$."

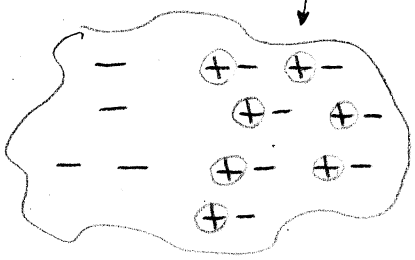
(a) Is Trevor correct? If not, correct his application of the number line model.

No way! $-4-7=-11$. Start at zero facing right. Back up 4. Turn around. Go forward 7.



(b) Use a different model to illustrate Trevor's problem.

Start with -4



Then take away 7 +'s (circled).

Leaves 11 $-$'s.

15. (3 points) Clearly state the rule for adding two integers with opposite signs. Give an example that illustrates your rule.

Subtract their absolute values, least from greatest. Then give the result the sign of the addend with the greatest absolute value.

eg. $2+(-5)=-(5-2)=-3$

16. (2 points) Use short division to compute $-284135\div(-5)$.

Neg $\div$ Neg = pos. So I'll compute $284135\div 5$

Final result will be +.

$$5\overline{)28^{3}4^{4}1^{1}3^{3}5}\quad 56827$$

Quotient is 56,827.

17. (1 point) Suppose a and b are integers. Which one of the following is equal to $-a-(-b)$?

(a) $-a+b$
(b) $a-b$
(c) $-a-b$
(d) $a+b$

18. (1 point) Which one of the following is a true statement?

(a) $0 \mid 7$
(b) $9 \mid 12321$
(c) $24 \mid 6 = 4$
(d) 18 divides 6

19. (1 point) What is the sign of $2 \times (-2) \times (-3) \div (-2) \times 5 \div (-1)$?

(a) positive
(b) negative

EQUALS +30

20. (1 point) Suppose $d \mid a$ and $d \mid b$. Which one of the following is NOT necessarily true?

(a) $d \mid (2a-3b)$
(b) $d \mid ab$
(c) $a \mid ad$ if $a \neq 0$
(d) $a \mid d$

21. (1 point) Suppose x is an integer. Which one of the following is equal to $-x \div (-2)$?

(a) $-x \div 2$
(b) $-(x \div 2)$
(c) $x \div (-2)$
(d) $x \div 2$

22. (1 point) The number 1 is also know as

(a) the multiplicative identity.
(b) the additive identity.
(c) the additive inverse.
(d) the multiplicative inverse.

23. (3 points) Explain why the algorithm illustrated below works. Then use it to compute 19×53.

THIS IS A PARTIAL PRODUCTS ALGORITHM. EACH TERM IS A PARTIAL PRODUCT. IT IS THE SAME AS USING FOIL ON $(20+4)(40+6)$ OR USING THE RECTANGLE:

$$\begin{array}{r} 24 \\ \times\ 46 \\ \hline 24 \\ 120 \\ 160 \\ 800 \\ \hline 1104 \end{array} \quad \begin{array}{l} \leftarrow 6\times 4 \\ \leftarrow 6\times 20 \\ \leftarrow 40\times 4 \\ \leftarrow 40\times 20 \end{array}$$

40	6	
800	120	20
160	24	4

Ex

$$\begin{array}{r} 19 \\ \times\ 53 \\ \hline 27 \\ 30 \\ 450 \\ +\ 500 \\ \hline 1007 \end{array}$$

$19 \times 53 = 1007$

24. (2 points) Use the standard long division algorithm to compute $54192 \div 24$.

$$\begin{array}{r} 2258 \\ 24\overline{)54192} \\ -48\downarrow \\ 61 \\ -48\downarrow \\ 139 \\ -120\downarrow \\ 192 \\ -192 \\ \hline \end{array}$$

$54192 \div 24 = 2258$

25. (2 points) Any number in which each digit except 0 appears exactly 3 times must be divisible by 3. Explain why this must be true and give an example of such a number.

SUCH A NUMBER IS 765567222756.

A NUMBER IS DIVISIBLE BY 3 IF AND ONLY IF THE SUM OF ITS DIGITS IS DIVISIBLE BY 3. IF EACH DIFFERENT DIGIT OCCURS 3 TIMES, THEN THE TOTAL SUM OF DIGITS WILL BE 3 TIMES THE SUM OF DISTINCT DIGITS, AND THEREFORE IT WILL BE A MULTIPLE OF 3.

IN MY EXAMPLE...

$7+6+5+5+6+7+2+2+2+7+5+6 = 7+7+7+6+6+6+5+5+5+2+2+2$

$= 3(7+6+5+2) =$ MULT OF 3.

26. (2 points) If 123 is divided by a number and the remainder is 13, what are the possible divisors?

$123 \div b = q \text{ r } 13 \Rightarrow 123 = bq + 13$

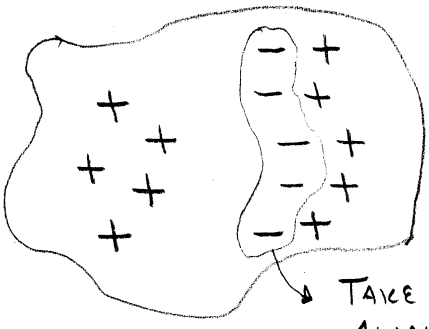
THE DIVISOR b MUST BE GREATER THAN 13.

$bq = 110 \rightarrow$ FACTORS OF 110 ARE $1, 2, 5, 10, 11, 22, 55, 110$

POSSIBLE VALUES OF b ARE 22, 55, 110

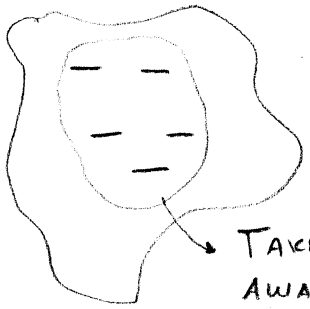
27. (3 points) It is common for students to make mistakes when computing differences such as $5 - (-5)$ and $-5 - (-5)$. Use the charged-field model to compute each difference. Be sure to label which is which.

$5-(-5)$ START WITH 5 +'s
TAKE AWAY 5 −'s



TAKE THESE AWAY... $5-(-5) = 10$

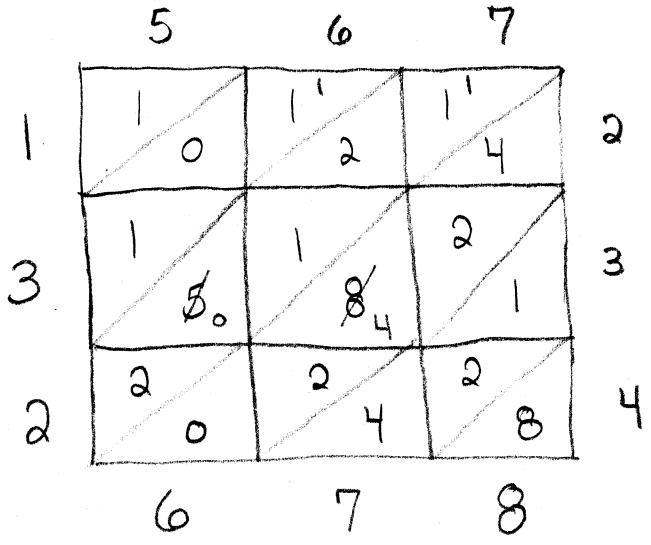
$-5-(-5)$ START WITH 5 −'s.
TAKE AWAY 5 −'s



TAKE THESE AWAY
$-5-(-5) = 0$

28. (2 points) Use a nonstandard multiplication algorithm to compute 567×234.

LATTICE...



132,678

29. (2 pts ex cred) See problem #8 on page 172. Use the Russian peasant algorithm to compute 85×93.

85 x 93 ...

Halves	Doubles
85	(93)
42	186 x 2
21	(372) x 2
10	744 x 2
5	(1488) x 2
2	2976 x 2
1	(5952)

1 3 1
5952
1488
372
+ 93
7905

85 x 93 = 7905