

Math 200 - 1st Final Exam

December 1, 2010

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary. Multiple choice problems are worth 0, 1, or 2 points depending on your answer and the work shown.

1. When using the 4-step, problem-solving process which one of these strategies would NOT be considered part of devising a plan?

- (a) Look for a pattern.
- (b) Sketch a picture.
- (c) Examine a smaller problem.
- (d) Determine what information is irrelevant.

2. Choose the subtraction model that best fits the following problem situation: *Caleb has 5 cookies and Katie has 3 cookies. How many more cookies does Caleb have?*

- (a) comparison
- (b) take away
- (c) number line
- (d) missing addend

3. Use any appropriate addition algorithm to compute the following sum.

$$33_{\text{four}} + 22_{\text{four}} + 32_{\text{four}} + 1_{\text{four}} + 20_{\text{four}} + 3_{\text{four}} + 13_{\text{four}}$$

- (a) 124_{four}
- (b) 130_{four}
- (c) 322_{four}
- (d) 333_{four}

SCRATCH →

$$\begin{array}{r} 3 \\ 23 \\ 22 \\ 32 \\ 1 \\ 20 \\ 3 \\ 13 \\ \hline 322 \end{array}$$

4. Which one of the following facts demonstrates the associative property of addition?

- (a) $2(5 + 1) + 3 = 2(1 + 5) + 3$
- (b) $(6x + 10) = 2(3x + 5)$
- (c) $(x + 3) + (2 + y) = (y + 2) + (x + 3)$
- (d) $(4 + 1) + 2 = 4 + (1 + 2)$

5. Which one of the following is not a basic property of the Hindu-Arabic numeration system?
- (a) Place value is based on powers of 10.
 - (b) Each digit in a numeral has a face value, a place value, and a value.
 - (c) Each number is either prime or composite.
 - (d) All numerals are constructed from the 10 digits.

6. Consider the following conjecture:

$$\text{If } 3 \mid x, \text{ then } 6 \mid x.$$

Which one of the following is a counterexample?

- (a) $3 \mid 18$ and $6 \mid 18$.
- (b) $3 \nmid 10$ and $6 \nmid 10$
- (c) $3 \mid 27$ and $6 \nmid 27$
- (d) The conjecture is true.

7. The prime factorization of a number is $2^5 \cdot 3^3 \cdot 5$. How many positive integer divisors does the number have?

- (a) 15
 - (b) 48
 - (c) 64
 - (d) 24
- $6 \times 4 \times 2 = 48$

8. Which one of these numbers is the 835th term of the following arithmetic sequence?

$$10, 13, 16, 19, 22, 25, \dots$$

- (a) 2512
 - (b) 2518
 - (c) 2515
 - (d) 2524
- $N^{\text{TH}} \text{ TERM} = 3N + 7$
- $835^{\text{TH}} \text{ TERM} = 3(835) + 7 =$

9. Which one of the following is an example of deductive reasoning?

- (a) Every elephant I have ever seen is gray. Therefore every elephant must be gray. ← INDUCTIVE
- (b) It has rained every day. Therefore it will rain tomorrow. ← INDUCTIVE
- (c) Since multiplication is commutative, $15 \cdot 37$ must be equal to $37 \cdot 15$.
- (d) A sequence begins with 1,2,3,4. The next term must be 5. ← INDUCTIVE

10. Choose the division model that best fits the following problem situation: 52 playing cards are shared equally among 4 people. How many cards does each person get?

- (a) partition
- (b) missing factor
- (c) repeated subtraction
- (d) division algorithm

How many in each group?

11. Let $A = \{0, 1, 2, 3\}$. Choose the set B for which all of the following are true?

$$4 \in B, \quad A \sim B, \quad A \cap B = \emptyset, \quad (7, 2) \in B \times A$$

- ~~(a) $B = \{4, 7\}$~~
- (b) $B = \{4, 5, 7, 8\}$
- ~~(c) $B = \{3.5, 4, 8, 9\}$~~
- ~~(d) $B = \{0, 2, 4, 7\}$~~

12. Convert 4103_{six} to base ten.

- (a) 5403
- (b) 903
- (c) 5418
- (d) 3555

$$4 \times 6^3 + 1 \times 6^2 + 0 \times 6 + 3 = 4 \times 216 + 36 + 3 = 903$$

13. (5 points) In the harbor there were ships with 3 masts and ships with 4 masts. John saw 7 ships and counted a total of 26 masts. How many ships had 3 masts and how many had 4? (Use any technique to solve the problem, but show your work or explain your reasoning. You will receive no credit for simply writing a answer.)

LET $x = \text{NUMBER OF 3-MAST SHIPS}$
 $y = \text{NUMBER OF 4-MAST SHIPS}$

$$\begin{aligned} x + y &= 7 & \rightarrow & y = 7 - x \\ 3x + 4y &= 26 & & \\ & & & \downarrow \\ & & & 3x + 4(7 - x) = 26 \\ & & & 3x + 28 - 4x = 26 \\ & & & -x = -2 \\ & & & x = 2 \end{aligned}$$

$$x = 2 \Rightarrow y = 7 - 2 = 5$$

2 SHIPS HAD 3 MASTS.
 5 SHIPS HAD 4 MASTS

↑ THAT MAKES 7 SHIPS AND 26 MASTS!

14. (1 point) Give an example of a geometric sequence.

$$2, 2 \cdot 5, 2 \cdot 5^2, 2 \cdot 5^3, 2 \cdot 5^4, \dots$$

START WITH 2,

MULT BY 5

$$2, 10, 50, 250, 1250, \dots$$

15. (5 points) Clearly state the rule for adding two integers with opposite signs. Give an example that illustrates your rule.

SUBTRACT ABSOLUTE VALUES, LEAST FROM GREATEST. GIVE THE RESULT THE SIGN OF THE INTEGER WITH THE GREATEST ABSOLUTE VALUE.

↙ sign of -9

$$\text{Ex. } 5 + (-9) = -(9 - 5) = -4$$

16. (5 points) Use a model to illustrate each of the following. (Model what is given, not a related problem.)

(a) $-5 + 3$

NUMBER LINE:

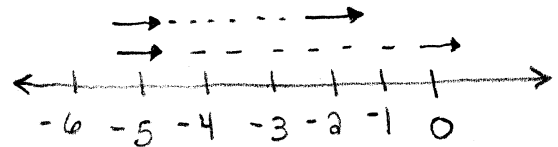
① START AT ZERO FACING RIGHT

② BACK UP 5

③ STAY FACING RIGHT

④ MOVE FORWARD 3

⑤ END AT $-2 \Rightarrow -5 + 3 = -2$



(b) $(-3) \times (-4)$

PATTERN: $3 \times (-4) = -4 + (-4) + (-4) = -12$

$2 \times (-4) = -4 + (-4) = -8$

$1 \times (-4) = -4$

$0 \times (-4) = 0$

PRODUCT INCREASES BY 4

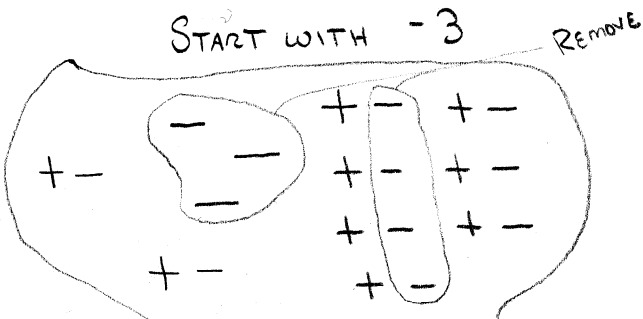
$(-1) \times (-4) = 4$

$(-2) \times (-4) = 8$

$(-3) \times (-4) = 12$

CHARGES:

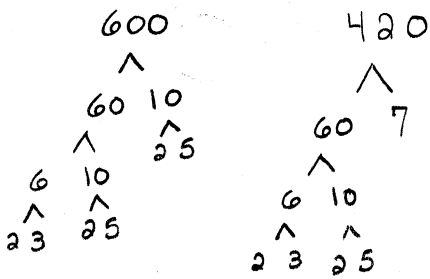
REMOVE 7 NEGATIVES.



LEFT WITH +4

$-3 - (-7) = +4$

17. (5 points) Find the prime factorizations of 600 and 420. Then find the GCD and LCM.



$$600 = 2^3 \cdot 3 \cdot 5^2$$

$$420 = 2^2 \cdot 3 \cdot 5 \cdot 7$$

$$\text{GCD} = 2^2 \cdot 3 \cdot 5 = 60$$

$$\text{LCM} = 2^3 \cdot 3 \cdot 5^2 \cdot 7 = 4200$$

18. (5 points) Let $A = \{1, 2, 3\}$ and $B = \{x, y\}$.

(a) Find the Cartesian product $A \times B$.

$$A \times B = \{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}$$

- (b) Write a word problem involving the multiplication fact $3 \times 2 = 6$ that is most naturally modeled using the Cartesian product model.

SALLY HAS 3 HATS AND 2 SCARVES. HOW MANY HAT/SCARF COMBINATIONS DOES SALLY HAVE?

19. (5 points) Classify each of the following as true or false. If false, provide a counterexample.

(a) If $n(A) = n(B)$, then $A = B$.

$$\text{FALSE, } A = \{1, 2\}, B = \{3, 4\}$$

(b) If $A - B = \emptyset$, then $A = B$.

$$\text{FALSE, } A = \{1, 2\}, B = \{1, 2, 3, 4\}$$

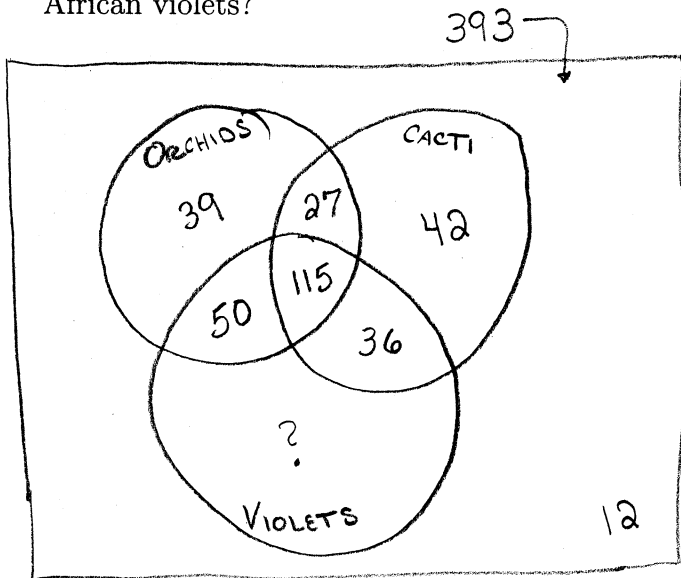
(c) If $A \subseteq B$, then $A \cap B = A$.

TRUE.

20. (5 points) In a survey of 393 horticulturists, the following information was obtained:

- 231 keep orchids;
- 220 keep cacti;
- 165 keep African violets and orchids;
- 151 keep African violets and cacti;
- 142 keep orchids and cacti;
- 115 keep all three types of plants; and
- 12 keep none of these types of plants.

Organize the data in a three-set Venn diagram. How many people surveyed keep only African violets?



$$39 + 27 + 42 + 50 + 115 + 36 + 12 = 321$$

$$\begin{aligned} \# \text{ OF VIOLET KEEPERS} &= 393 - 321 \\ &= \boxed{72} \end{aligned}$$

21. (5 points) Find digits s and t so that $1s52t$ is divisible by 12, and $1s52t$ has the greatest possible value. Once you've found your number, also test it for divisibility by 8 and 9.

Divisible by 12 \Rightarrow

Divisible by 3 & 4

By 4: $2t$ must be divisible

by 4 so $t = 0, 4, \text{ or } 8$

By 3: $1 + s + 5 + 2 + t = 8 + s + t$
must be divisible by 3

S must be as big as possible

$s = 9$ is possible if $t = 4$

Number must be

$\boxed{19524}$

Sum of digits = 21 \Rightarrow Not divisible by 9

$8 \nmid 524 \Rightarrow$ Not divisible by 8

22. (5 points) State the four steps of the 4-step, problem-solving process. Choose any one of your steps (Tell which you choose!) and list two strategies associated with that step.

- ① UNDERSTAND THE PROBLEM
- ② DEVISE A PLAN
- ③ CARRY OUT THE PLAN
- ④ LOOK BACK

STRATEGIES FOR LOOK BACK:

i) CHECK YOUR ANSWER
IN THE ORIGINAL
WORDING OF THE
PROBLEM

ii) GENERALIZE

23. (5 points)

(a) What does it mean for a positive integer to be prime?

A POS. INTEGER IS PRIME IF IT HAS EXACTLY
TWO DISTINCT POSITIVE INTEGER FACTORS.

(b) When testing to determine whether 1073 is prime, what is the greatest prime that must be checked?

$\sqrt{1073} \approx 32.75 \Rightarrow 31$ IS THE GREATEST PRIME
THAT MUST BE CHECKED.

(c) Determine whether 1073 is prime. If it is not prime, determine its prime factorization.

$31 \nmid 1073$

$29 \mid 1073$

$1073 = 29 \times 37$

(d) What does it mean for a positive integer to be composite?

A POS. INTEGER OTHER THAN 1 IS COMPOSITE IF
IT IS NOT PRIME.