

Math 200 - 2nd Final Exam

December 15, 2010

Name key
Score _____

Show all work to receive full credit. Supply explanations where necessary. Multiple choice problems are worth 0, 1, or 2 points depending on your answer and the work shown.

1. Choose the division model that best fits the following problem situation: *Sammy's mother made 36 cookies for the school bake sale. She would like to put them into bags that hold 4 cookies each. How many bags does she need?*

- (a) partition
- (b) missing factor
- (c) repeated subtraction
- (d) Cartesian product

↑ How many groups?

2. Which one of the following facts illustrates the commutative property of multiplication?

- (a) $(2 + x) + y = y + (2 + x)$
- (b) $3 \cdot (x + 7) + 2 = 3x + 23$
- (c) $3(2)(x + y) = 2(3)(x + y)$
- (d) $(2x + 3y) + 8 = 2x + (3y + 8)$

3. What is the value of the digit 5 in the base-eight numeral 6543_{eight} ?

- (a) 5
- (b) $5 \cdot 64$
- (c) 500
- (d) $5 \cdot 512$

$$5 \cdot 8^2 = 5 \cdot 64$$

4. Suppose $A = \{x, y, z\}$, $B \subseteq A$, and $n(B) = 3$. Which one of the following must be true?

- (a) $(x, x) \in A \times B$
- (b) $A \cap B = \emptyset$
- (c) $n(A \cup B) = 6$
- (d) $B - A = 3 - x - y - z$

IT MUST BE TRUE THAT $A = B$.

5. Which one of these fractions is in lowest terms?

- (a) $6/111 = 2/37$
- (b) $81/35$
- (c) $51/85 = 3/5$
- (d) $5/40 = 1/8$

6. Choose the multiplication model that best fits the following problem situation: *The graduates walked into the auditorium as a group in ten rows of four. How many graduates were there?*

- (a) Cartesian product
- (b) repeated addition
- (c) area/array
- (d) set partition

7. Convert the base-ten numeral 372 to base-four.

- (a) 11310_{four}
- (b) 78_{four}
- (c) 1131_{four}
- (d) 1032_{four}

$$\begin{aligned}
 4^0 &= 1 \\
 4^1 &= 4 \\
 4^2 &= 16 \\
 4^3 &= 64 \\
 4^4 &= 256
 \end{aligned}$$

$$\begin{array}{r}
 4^4 = 256 \overline{) 372} \\
 \underline{- 256} \\
 116 \\
 64 \\
 \underline{ 64} \\
 52 \\
 48 \\
 \underline{ 48} \\
 4 \\
 \underline{ 4} \\
 0
 \end{array}$$

8. When using the 4-step, problem-solving process which one of these strategies would NOT be considered part of understanding the problem?

- (a) Reread the problem.
- (b) State the problem in your own words.
- (c) Determine what information is not needed.
- (d) Write an equation. ← DEVISE A PLAN

9. Suppose $A = 2^3 \cdot 5^2 \cdot 7 \cdot 13^3$ and $B = 2 \cdot 3 \cdot 5^3 \cdot 13^2$. Find the LCM of A and B .

- (a) $2 \cdot 3 \cdot 5^2 \cdot 7 \cdot 13^2$
- (b) $2^3 \cdot 3 \cdot 5^3 \cdot 7 \cdot 13^3$
- (c) $2 \cdot 5^2 \cdot 13^2$
- (d) $2^4 \cdot 3 \cdot 5^5 \cdot 7 \cdot 13^5$

↑
EACH FACTOR MAX # OF TIMES

$$LCM = 2^3 \cdot 3 \cdot 5^3 \cdot 7 \cdot 13^3$$

10. Which one of these numbers is the 1371st term of the following arithmetic sequence?

$$\begin{array}{ccccccc}
 18, & 25, & 32, & 39, & 46, & 53, & 60, & 67, & \dots \\
 \downarrow & \downarrow & \downarrow & & & & & & \\
 7 & 7 & 7 & & & & & &
 \end{array}$$

- (a) 9608
- (b) 9615
- (c) 9601
- (d) 9597

$$N^{\text{TH}} \text{ TERM} = 7N + 11$$

$$7(1371) + 11 = 9608$$

11. Which one of the following is an example of inductive reasoning?

(a) $2(3 + 5) = 2(5 + 3)$

GENERALIZATION BASED ON OBSERVATION.

(b) A sequence begins with 2,4,6,8. The next term must be 10.

(c) If $x = 10$, then $2x + 3 = 23$.

(d) Wednesdays are pizza days, so today is a pizza day.

12. Let W be the set of all whole numbers. The set A is defined below using set-builder notation. Which one of the given sets is equal to A ?

$$A = \{x \mid x = 2n \text{ where } n \in W \text{ and } n < 2\}$$

(a) $\{0, 2\}$

(b) $\{0, 1\}$

(c) $\{0, 2, 4, 6, \dots\}$

(d) $\{\dots, -6, -4, -2, 0, 2\}$

13. Compute $\frac{81}{20} \div \frac{60}{36}$ and write your answer in lowest terms.

(a) $81/13$

(b) $243/100$

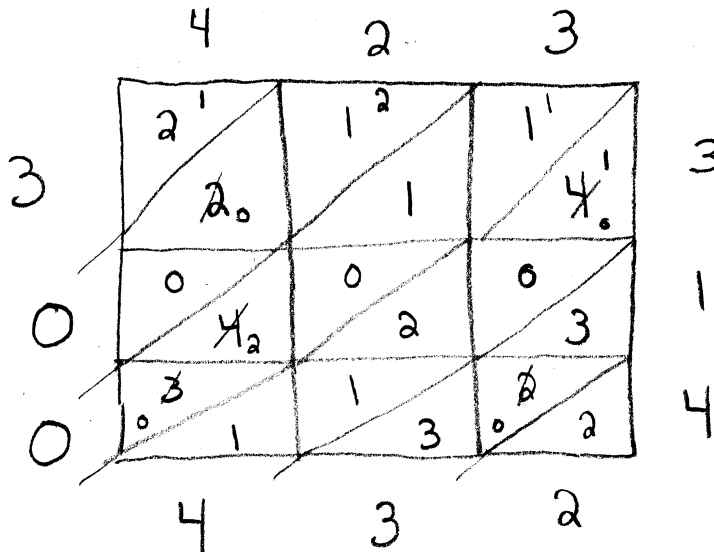
(c) $27/4$

(d) $3/1600$

$$\frac{81}{20} \times \frac{36}{60} = \frac{243}{100}$$

9 3

14. (5 points) Use any multiplication algorithm to compute $423_{\text{five}} \times 314_{\text{five}}$.



300432 FIVE

15. (5 points) Test the number 749968830 for divisibility by ~~2~~, ~~3~~, ~~4~~, ~~5~~, ~~6~~, ~~8~~, ~~9~~, ~~10~~, and 20. Show work and/or explain your reasoning.

DIVISIBLE BY 10 BECAUSE ITS ONES DIGIT IS ZERO.

DIVISIBLE BY 2 & 5 BECAUSE ITS DIVISIBLE BY 10.

NOT DIVISIBLE BY 4 BECAUSE $4 \nmid 30$ (LOOK AT LAST TWO DIGITS!)

NOT DIVISIBLE BY 8 OR 20 BECAUSE NOT DIVISIBLE BY 4.

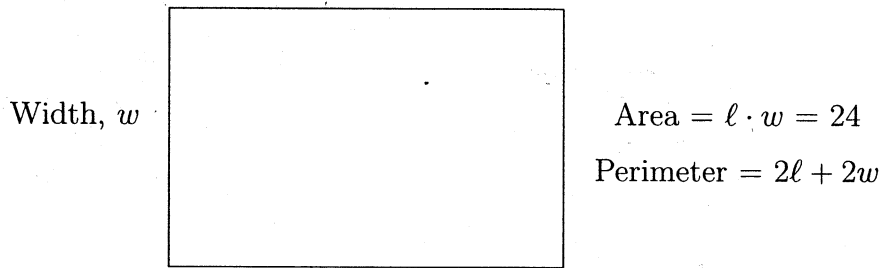
SUM OF DIGITS IS $7+4+9+9+6+8+8+3 = 54$

DIVISIBLE BY 3 AND 9 BECAUSE $3 \mid 54$ AND $9 \mid 54$.

DIVISIBLE BY 6 BECAUSE DIVISIBLE BY BOTH 2 & 3

Summary
 Divis. by
 2, 3, 5, 6,
 9, 10

16. (5 points) The area of a rectangle is 24 square inches. Its length and width are natural numbers. Use this information to find the rectangle with the least possible perimeter.



LENGTH & WIDTH

ARE NATURAL NUMBERS

AND THEIR PRODUCT IS

24. THE ONLY POSSIBILITIES

ARE IN THE TABLE ...

Length, l

l	w	lw	PERIMETER
1	24	24	$2 + 48 = 50$ IN
2	12	24	$4 + 24 = 28$ IN
3	8	24	$6 + 16 = 22$ IN
4	6	24	$8 + 12 = 20$ IN
6	4	24	$12 + 8 = 20$ IN
8	3	24	$16 + 6 = 22$ IN
12	2	24	$24 + 4 = 28$ IN
24	1	24	$48 + 2 = 50$ IN

RECTANGLE

WITH LEAST PERIMETER

MEASURES

4 IN x 6 IN OR 6 IN x 4 IN.

17. (5 points)

- (a) Find the fourth term of the geometric sequence whose first term is 3 and whose ratio is 5.

$$3, 15, 75, \boxed{375}, 1875, 9375, \dots$$

- (b) A recursive sequence is defined as follows:

$$B_1 = -2, \quad B_n = -3 \cdot B_{n-1} + 5, \text{ for } n = 2, 3, 4, \dots$$

Find the third term of the sequence.

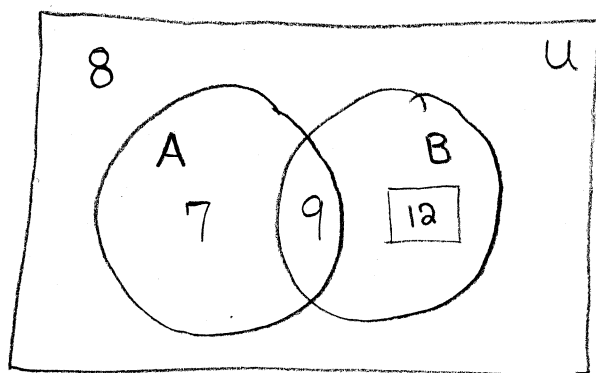
$$B_1 = -2, \quad B_2 = -3B_1 + 5 = -3(-2) + 5 = 11$$

$$B_3 = -3B_2 + 5 = -3(11) + 5 = \boxed{-28}$$

- (c) Find the next term: 2, 4, 9, 17, 28, 42, ... $42 + 17 = \boxed{59}$

$$\begin{array}{cccccc} \vee & \vee & \vee & \vee & \vee & \vee \\ 2 & 5 & 8 & 11 & 14 & 17 \end{array}$$

18. (5 points) Suppose A and B are subsets of U , and U has 36 elements. Use a two-set Venn diagram to help you determine $n(B)$ if $n(A) = 16$, $n(A \cap B) = 9$, and $n(\overline{A \cup B}) = 8$.



36 ELEMENTS
INSIDE RECTANGLE

$$36 - (8 + 7 + 9) = 12$$

$$\boxed{N(B) = 21}$$

19. (5 points) Use a model to illustrate and compute each product. (Model what is given, not a related problem.)

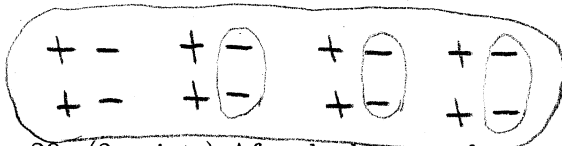
$$(a) 3 \times \frac{1}{2} = \underbrace{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}}_{\text{REPEATED ADDITION}} = \frac{1+1+1}{2} = \frac{3}{2}$$

(b) $\frac{1}{2} \times \frac{3}{4}$

$\frac{1}{2}$ OF $\frac{3}{4} = \frac{3}{8}$

(c) $-3 \times (-2)$

START WITH ZERO



TAKE OUT
3 groups of
2 NEGATIVES
LEFT WITH 6 POSITIVES
 $-3 \times (-2) = +6$

20. (2 points) After looking at these examples:

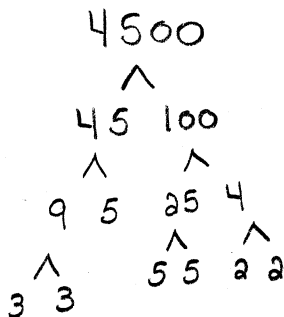
$$5 \cdot 3 + 5 \cdot 5 = 40, \quad 5 \cdot 2 + 5 \cdot 4 = 30, \quad 5 \cdot 8 + 5 \cdot 10 = 90,$$

Marcus conjectured that the sum of two multiples of 5 is a multiple of 10. Is he correct? If not, give a counterexample.

NOT CORRECT $5 \cdot 3 + 5 \cdot 2 = 15 + 10 = 25$

AND 25 IS NOT A
MULTIPLE OF 10.

21. (5 points) Find the prime factorization of 4500. Then use your factorization to determine the number of positive integer divisors of 4500.



NUMBER OF DIVISORS IS

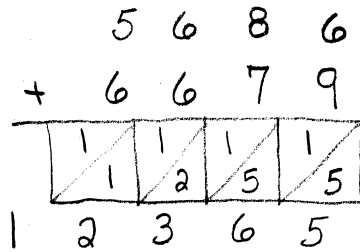
$$3 \times 3 \times 4 = \boxed{36}$$

$$\boxed{4500 = 2^2 \cdot 3^2 \cdot 5^3}$$

22. (5 points) Use a NONSTANDARD algorithm to compute each of the following.

(a) $5686 + 6679$

LATTICE
ALG.



$12,365$

(b) $345 - 269$

EQUAL
ADDITION
ALG.

$$\begin{array}{r} 345 + 1 \\ - 269 + 1 \\ \hline \end{array} \Rightarrow \begin{array}{r} 346 + 30 \\ - 270 + 30 \\ \hline \end{array} \Rightarrow \begin{array}{r} 376 \\ - 300 \\ \hline 76 \end{array}$$

76

(c) $6745 \div 5$

SHORT
DIVISION

$$5 \overline{) 6745}$$

1349

23. (2 points) State a basic property of the Hindu-Arabic numeration system.

ALL NUMERALS ARE CONSTRUCTED FROM 10 BASIC DIGITS.

24. (1.5 points) In a sentence or two, describe how to find $9/8$ on the number line.

DIVIDE THE INTERVAL FROM 0 TO 1 INTO 8 EQUAL-SIZED PIECES.

STARTING AT ZERO, MOVE TO THE RIGHT A LENGTH CORRESPONDING TO 9 OF THOSE PIECES.

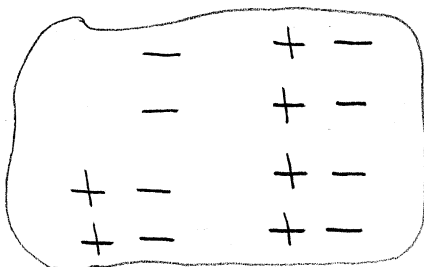
25. (1.5 points) Write the expanded form of the number 54967.

$$5 \times 10^4 + 4 \times 10^3 + 9 \times 10^2 + 6 \times 10^1 + 7 \times 10^0$$

26. (2 points) Use an integer subtraction model to illustrate and compute $-2 - (-5)$.

$-2 - (-5) = +3$

START WITH -2



NOW TAKE AWAY

5 NEGATIVES

TO GET

7

+3

