Math 200 - Test 1
February 15, 2010

Name key

Score

Show all work to receive full credit (even on multiple-choice problems). Supply explanations when necessary. Problems marked PSP are problem-solving problems. When solving the PSP's, you should provide evidence that you have used the 4-step problem-solving process.

1. (4 points) Clearly state the four steps of the problem-solving process (in order).

   (1) Understand the problem

   (2) Devise a plan

   (3) Carry out the plan

   (4) Look back

2. (3 points) State three different strategies for understanding the problem.

   See out textbook, page 4.

3. (1 point) When using the 4-step, problem-solving process which one of these strategies would NOT be considered part of devising a plan?

   (a) Look for a pattern.
   (b) Determine what is known and unknown. ← Part of understand prob.
   (c) Work backward.
   (d) Guess and check.

4. (3 points) (PSP) Explain why the following problem has no solution: Find three consecutive odd numbers whose sum is 102.

Consecutive odds differ by 2, e.g. 19, 21, 23.

Let's think about some sums near 102...

$31 + 33 + 35 = 99$

$33 + 35 + 37 = 105$

→ Any other consecutive odds must add up to a number less than 99 or greater than 105.

Another approach:

The sum of two odds is an even.

The sum of an even & an odd is odd.

Therefore, the sum of three odds must be odd.

5. (3 points) A sequence is defined recursively as follows:

$$A_1 = 4 \quad \text{and} \quad A_n = A_{n-1} + 3 \quad \text{for } n = 2, 3, 4, \ldots$$

(a) Find the first five terms of the sequence.

$A_1 = 4$

$A_2 = 4+3 = 7$

$A_3 = 7+3 = 10$

$A_4 = 10+3 = 13$

$A_5 = 13+3 = 16$

$\boxed{4, 7, 10, 13, 16, \ldots}$

(b) In addition to calling this a recursive sequence, we have another name for this type of sequence. What is it?

THE SEQUENCE IS ARITHMETIC.

(DIFFERENCE IS 3)

6. (3 points) There is an old Indian story of a poor wise man who tricked the king into giving him some rice. The king told the man he would give him 1 grain of rice on day one, 2 grains on day two, 4 grains on day three, 8 grains on day four, and so on for a month.

(a) Write down the first eight terms in the sequence of the numbers of grains of rice. What is the name of this type of sequence?

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5	DAY 6	DAY 7	DAY 8
1,	2,	4,	8,	16,	32,	64,	128

THIS IS A GEOMETRIC SEQUENCE

(RATIO IS 2)

(b) Find a formula for the nth term of the sequence.

N^{TH} TERM $= 1 \cdot 2^{N-1}$ OR JUST N^{TH} TERM $= 2^{N-1}$

7. (1 point) Which of one of these mathematicians is associated with the following famous sequence?

$$\underbrace{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots}_{\text{FIBONACCI SEQUENCE}}$$

(a) Bernhard Riemann

(b) Gerolamo Cardano

(c) Fibonacci

(d) Carl Friedrich Gauss

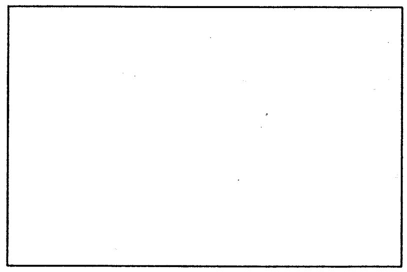
8. (4 points) (PSP) The area of a rectangle is 24 square inches. Its length and width are natural numbers. Use this information to find the rectangle with the least possible perimeter.

SINCE THE AREA IS FIXED, AND THE LENGTH AND WIDTH ARE COUNTING NUMBERS, I SHOULD BE ABLE TO MAKE A TABLE SHOWING ALL POSSIBLE LENGTHS AND WIDTHS.

I NEED TO MAKE SURE THAT $\ell \times w = 24$.

I NEED TO LOOK AT ALL FACTORIZATIONS OF 24.

Width, w



Length, ℓ

Area $= \ell \cdot w = 24$

Perimeter $= 2\ell + 2w$

THIS SEEMS TO MAKE GOOD SENSE, BECAUSE THESE DIMENSIONS MAKE THE RECT MOST LIKE A SQUARE.

	LENGTH	WIDTH	PERIMETER
	1	24	2 + 48 = 50 IN
	2	12	4 + 24 = 28 IN
	3	8	6 + 16 = 22 IN
*	4	6	8 + 12 20 IN
*	6	4	20 IN (SAME AS 4x6)
	8	3	22 IN (SAME AS 3x8)
	12	2	28 IN (SAME AS 2x12)
	24	1	50 IN (SAME AS 1x24)

THE DIMENSIONS THAT GIVE THE LEAST PERIMETER ARE 4 IN X 6 IN.

9. (2 points) After looking at these examples:

$$3 \cdot 8 + 3 \cdot 2 = 30, \quad 3 \cdot 3 + 3 \cdot 5 = 24, \quad 3 \cdot 6 + 3 \cdot 8 = 42,$$

Jolie conjectured that the sum of two multiples of 3 is always a multiple of 6. Is she correct? If not, give a counterexample.

SHE IS NOT CORRECT.

$3 \cdot 2 + 3 \cdot 3 = 6 + 9 = 15$, AND 15 IS NOT A MULTIPLE OF 6.

10. (3 points) The first difference of a sequence is $3, 6, 9, 12, 15, \ldots$. The first two terms of the original sequence add up to 7. Find the first six terms of the original sequence.

THE FIRST TWO TERMS ADD UP TO 7

AND THEIR DIFFERENCE IS 3 $\Longrightarrow$ THE TERMS MUST BE 2 & 5

$$\boxed{2,\ 5,\ 11,\ 20,\ 32,\ 47,\ \ldots}$$

$$3 \quad 6 \quad 9 \quad 12 \quad 15$$

11. (3 points) Which one of these numbers is the 371st term of the following arithmetic sequence? (Make sure you show your work.)

$$4, 11, 18, 25, 32, 39, \ldots$$

$$7 \quad 7 \quad 7 \quad 7 \quad 7$$

(a) 2594
(b) 2597
(c) 2601
(d) 2587

N^{TH} TERM IS $7N - 3$

371ST TERM IS $7(371) - 3$

$= 2594$

12. (2 points) Write the following set in roster (listing) notation.

$$\{x \mid x = 3n + 2,\ n \in \mathbb{N}\}$$

$$= \{5, 8, 11, 14, 17, 20, \ldots\}$$

13. (1 point) What does it means for two sets to be equivalent?

TWO SETS ARE EQUIVALENT IF THERE IS A ONE-TO-ONE CORRESPONDENCE BETWEEN THEM.

14. (3 points) There are 138 terms in the following sequence. Find the sum of the terms.

$$5, 13, 21, 29, \ldots, 1093, 1101$$

(a) 151, 938

(b) 75, 969

(c) 605, 550

(d) 76, 314

$$2S = \begin{cases} 5 + 13 + 21 + \cdots + 1093 + 1101 \\ 1101 + 1093 + 1085 + \cdots + 13 + 5 \end{cases}$$

138 PAIRS OF 1106

$$S = \frac{1}{2}(138)(1106)$$

15. (2 points) Let $A = \{12, 14, 16, 18, \ldots, 50, 52\}$. Write the set A in set-builder notation.

$A = \{ x \mid x$ IS EVEN AND $12 \le x \le 52 \}$

16. (2 points) List all the subsets of $\{a, b\}$. THERE ARE 4 OF THEM.

$\{\ \}$ OR $\emptyset$, $\{a\}$, $\{b\}$, $\{a, b\}$

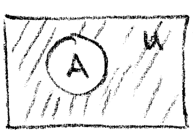
17. (1 point) Give an example of a set A with the property that $n(A) = 0$.

ONLY SUCH SET IS THE EMPTY SET

$\{\ \}$ OR $\emptyset$

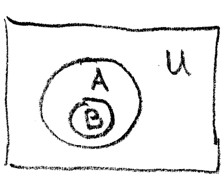
18. (2 points) Suppose that A and B are subsets of the universal set U. Use a Venn diagram to illustrate each of the following.

(a) $\overline{A}$



$\overline{A}$ IS SHADED

(b) $B \subseteq A$



$B \subseteq A$

19. (1 point) The formula for the nth triangular number is $(n^2 + n)/2$. Find the 7th triangular number.

7^{TH} TRIANGULAR # $= (7^2+7)/2$

$= (49+7)/2 = 56/2 = \boxed{28}$

20. (1 point) Give an example of a set A with the property that $\{2, 3\} \in A$.

$A = \{ \underbrace{\{1\}, \{2,3\}, \{2\}}_{\{2,3\} \text{ IS AN ELEMENT OF } A.} \}$

21. (1 point) Let U be the set of all PSC students, and let M be the set of all PSC math students. Describe an element of the set $\overline{M}$.

AN ELEMENT OF $\overline{M}$ IS A PSC STUDENT WHO IS NOT A MATH STUDENT.

22. (2 points) Give an example of a single set B that satisfies each of the following:

$$B \subseteq \mathbb{N}, \qquad B \sim \{x, y, z\}, \qquad 7 \in \overline{B}$$

$B = \{1, 2, 3\}$

23. (2 points) Find two terms that continue a possible pattern:

$2, 5, 12, 23, 38, \ldots$ $\boxed{57, \quad 80}$

3 7 11 15 19 23

4 4 4, 4 4