Math 200 - Test 1 Name ____________________

February 15, 2010 Score __________

Show all work to receive full credit (even on multiple-choice problems). Supply explanations when necessary. Problems marked PSP are problem-solving problems. When solving the PSP's, you should provide evidence that you have used the 4-step problem-solving process.

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1. (4 points) Clearly state the four steps of the problem-solving process (in order).

2. (3 points) State three different strategies for understanding the problem.

3. (1 point) When using the 4-step, problem-solving process which one of these strategies would NOT be considered part of devising a plan?

   (a) Look for a pattern.

   (b) Determine what is known and unknown.

   (c) Work backward.

   (d) Guess and check.

4. (3 points) (PSP) Explain why the following problem has no solution: Find three consecutive odd numbers whose sum is 102.

5. (3 points) A sequence is defined recursively as follows:

$$A_1 = 4 \quad \text{and} \quad A_n = A_{n-1} + 3 \quad \text{for } n = 2, 3, 4, \ldots$$

(a) Find the first five terms of the sequence.

(b) In addition to calling this a recursive sequence, we have another name for this type of sequence. What is it?

6. (3 points) There is an old Indian story of a poor wise man who tricked the king into giving him some rice. The king told the man he would give him 1 grain of rice on day one, 2 grains on day two, 4 grains on day three, 8 grains on day four, and so on for a month.

(a) Write down the first eight terms in the sequence of the numbers of grains of rice. What is the name of this type of sequence?

(b) Find a formula for the nth term of the sequence.

7. (1 point) Which of one of these mathematicians is associated with the following famous sequence?

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$$

(a) Bernhard Riemann

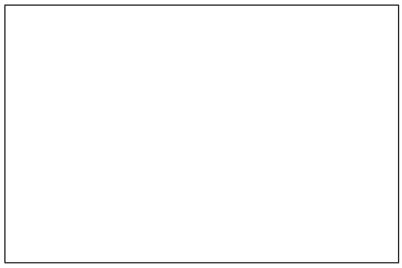
(b) Gerolamo Cardano

(c) Fibonacci

(d) Carl Friedrich Gauss

8. (4 points) (PSP) The area of a rectangle is 24 square inches. Its length and width are natural numbers. Use this information to find the rectangle with the least possible perimeter.

Width, w



$$\text{Area} = \ell \cdot w = 24$$
$$\text{Perimeter} = 2\ell + 2w$$

Length, ℓ

9. (2 points) After looking at these examples:

$$3 \cdot 8 + 3 \cdot 2 = 30, \quad 3 \cdot 3 + 3 \cdot 5 = 24, \quad 3 \cdot 6 + 3 \cdot 8 = 42,$$

Jolie conjectured that the sum of two multiples of 3 is always a multiple of 6. Is she correct? If not, give a counterexample.

10. (3 points) The first difference of a sequence is $3, 6, 9, 12, 15, \ldots$. The first two terms of the original sequence add up to 7. Find the first six terms of the original sequence.

11. (3 points) Which one of these numbers is the 371st term of the following arithmetic sequence? (Make sure you show your work.)

$$4, 11, 18, 25, 32, 39, \ldots$$

   (a) 2594
   (b) 2597
   (c) 2601
   (d) 2587

12. (2 points) Write the following set in roster (listing) notation.

$$\{x \mid x = 3n + 2,\ n \in \mathbb{N}\}$$

13. (1 point) What does it means for two sets to be equivalent?

14. (3 points) There are 138 terms in the following sequence. Find the sum of the terms.

$$5, 13, 21, 29, \ldots, 1093, 1101$$

(a) $151,938$
(b) $75,969$
(c) $605,550$
(d) $76,314$

15. (2 points) Let $A = \{12, 14, 16, 18, \ldots, 50, 52\}$. Write the set A in set-builder notation.

16. (2 points) List all the subsets of $\{a, b\}$.

17. (1 point) Give an example of a set A with the property that $n(A) = 0$.

18. (2 points) Suppose that A and B are subsets of the universal set U. Use a Venn diagram to illustrate each of the following.

(a) $\overline{A}$

(b) $B \subseteq A$

19. (1 point) The formula for the nth triangular number is $(n^2 + n)/2$. Find the 7th triangular number.

20. (1 point) Give an example of a set A with the property that $\{2, 3\} \in A$.

21. (1 point) Let U be the set of all PSC students, and let M be the set of all PSC math students. Describe an element of the set $\overline{M}$.

22. (2 points) Give an example of a single set B that satisfies each of the following:

$$B \subseteq \mathbb{N}, \qquad B \sim \{x, y, z\}, \qquad 7 \in \overline{B}$$

23. (2 points) Find two terms that continue a possible pattern:

$$2, 5, 12, 23, 38, \ldots$$