Math 200 - Test 3

April 21, 2010

Name key

Score _____

Show all work. Supply explanations when necessary.

1. (1 point) What algorithm is being illustrated here?

$$\begin{array}{r} 93 \\ -\,27 \\ \hline \end{array} \longrightarrow \begin{array}{r} 93 + 3 \\ -(27 + 3) \\ \hline \end{array} \longrightarrow \begin{array}{r} 96 \\ -\,30 \\ \hline 66 \end{array}$$

   (a) Scratch subtraction algorithm
   (b) Fast subtraction algorithm
   (c) Equal-additions algorithm
   (d) Nice-numbers algorithm

2. (1 point) Which one of the following does NOT mean the same as $a \mid b$?

   (a) a is divisible by b. a | b means b is divisible by a.
   (b) b is a multiple of a.
   (c) a is a factor of b.
   (d) There exists a unique integer k such that $b = ak$.

3. (1 point) What is the additive inverse of the expression $-3x + 2y - 8$?

   (a) $1/(-3x + 2y - 8)$
   (b) $3x + 2y - 8$
   (c) 0
   (d) $3x - 2y + 8$

4. (2 points) Compute the following sum: $143_{\text{five}} + 244_{\text{five}} + 313_{\text{five}} + 342_{\text{five}}$

   (a) 2202_{five}
   (b) 1042_{five}
   (c) 1212_{five}
   (d) 3212_{five}

   Scratch base 5

$$\begin{array}{r} {\scriptstyle 3\ \ 2} \\ 1\ 4_1\ 3 \\ 2_1\ 4_0\ 4_2 \\ 3\ 1\ 3_0 \\ 3_2\ 4_0\ 2 \\ \hline 2\ 2\ 0\ 2 \end{array}$$

5. (1 point) Which one of the following integers is NOT prime?

   (a) 991
   (b) 47
   (c) 19
   (d) 1

6. (4 points) In each case, use a different strategy to estimate the sum and give the name of your strategy.

(a) $453 + 397 + 405 + 367 + 398 + 421 + 347$

CLUSTERING. $\approx 7 \times 400 = 2800$

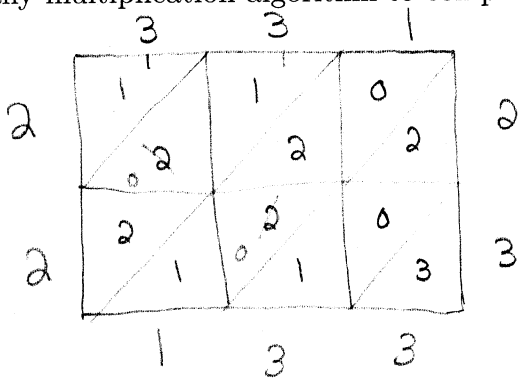
(b) $105 + 478 + 690 + 342 + 821$

ROUND TO NEAREST HUNDRED. $\approx 100 + 500 + 700 + 300 + 800 = 2400$

(c) $12 + 69 + 49 + 80 + 54 + 35 + 90$

MAKING COMPATIBLE NUMBERS. $\approx 10 + 90 + 70 + 30 + 50 + 50 + 80 = 380$

7. (4 points) Use any multiplication algorithm to compute $331_{\text{four}} \times 23_{\text{four}}$.



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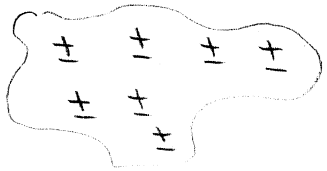
8. (3 points) In each case, use a different multiplication model to model and find the product.

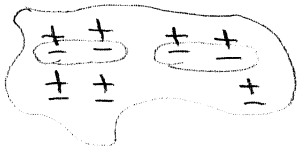
(a) $-2 \cdot (-2) = +4$

CHARGED FIELD:

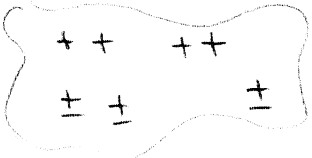
① START WITH ZERO-CHARGE FIELD



② TAKE OUT 2 GROUPS OF 2 NEGATIVES



③ LEAVES FIELD W/ +4



(b) $3 \cdot (-5) = -15$

PATTERN:

$3 \cdot 2 = 6$
$3 \cdot 1 = 3$
$3 \cdot 0 = 0$
$3 \cdot (-1) = -3$

$3 \cdot (-2) = -6$
$3 \cdot (-3) = -9$
$3 \cdot (-4) = -12$
$3 \cdot (-5) = -15$

9. (1 point) Which one of the following is a true statement?

(a) $4 \mid 2 = 2$

(b) 10 is divisible by 20

(c) $3 \mid 70002$

(d) $0 \mid 1$

10. (1 point) Which one of the following is equal to $-3 - 5$?

(a) $-3 + 5$

(b) $-3 + (-5)$

(c) $3 + 5$

(d) $3 + (-5)$

11. (1 point) Which of these is NOT a mental addition strategy?

(a) Adding from left to right

(b) Breaking up and bridging

(c) Using compatible numbers

(d) Clustering ← Estimation technique

12. (1 point) Suppose x is not zero. Which one of the following can NEVER be true?

(a) $|x| = -x$

(b) $|-x| = x$

(c) $|x| = |-x|$

(d) $|x| = -|x|$

13. (1 point) What is the greatest prime number you must consider to test whether 5669 is prime?

(a) 73

(b) 75 ← Not prime

(c) 911

(d) 23

$\sqrt{5669} \approx 75$

14. (1 point) Find the smallest positive integer that is divisible by three different primes.

(a) 6

(b) 30

(c) 8

(d) 105

$2 \cdot 3 \cdot 5 = 30$

15. (3 points) Clearly state the rule for adding two integers with opposite signs. Give an example that illustrates your rule.

SUBTRACT THEIR ABSOLUTE VALUES, LEAST FROM GREATEST.

GIVE YOUR RESULT THE SIGN OF THE ADDEND WITH THE GREATEST ABSOLUTE VALUE.

Ex $-5+2 = -(5-2) = -3$

16. (2 points) To square a number means to multiply it by itself. Carefully explain why the square of a number cannot be negative.

3 CASES:

① IF THE NUMBER IS POSITIVE, A POS TIMES A POS IS A POS (NOT NEGATIVE).

② IF THE NUMBER IS ZERO, 0×0 IS ZERO (NOT NEGATIVE)

③ IF THE NUMBER IS NEGATIVE, A NEG TIMES A NEG IS A POS (NOT NEGATIVE)

17. (4 points) Test the following number for divisibility by 2, 3, 4, 5, 6, 8, 9, and 10.

1273117679540520

D BY 2: YES. LAST DIGIT IS ZERO & $2 \mid 0$.

D BY 3: YES. DIGITS ADD UP TO 60 & $3 \mid 60$.

D BY 4: YES. LAST TWO DIGITS MAKE 20 & $4 \mid 20$.

D BY 5: YES. LAST DIGIT IS 0 & $5 \mid 0$.

D BY 6: YES. DIVISIBLE BY BOTH 2 & 3.

D BY 8: YES. LAST THREE DIGITS MAKE 520 AND $\frac{520}{8} = 65$

D BY 9: NO. DIGITS ADD UP TO 60 AND $9 \nmid 60$.

D BY 10: YES. LAST DIGIT IS ZERO & $10 \mid 0$.

18. (1 pt extra cred) Test the number above for divisibility by 11.

YES. $(1+7+1+7+7+5+0+2) - (2+3+1+6+9+4+5+0)$

$= 30 - 30 = 0$ AND $11 \mid 0$.

19. (3 points) Explain why the algorithm illustrated below works. Then use it to compute $9076 + 4689$.

$$\begin{array}{r} 568 \\ +757 \\ \hline 1200 \\ 110 \\ 15 \\ \hline 1325 \end{array}$$

THIS IS A LEFT-TO-RIGHT PARTIAL SUMS ALGORITHM.

WE'RE ADDING AT PARTICULAR PLACE VALUES FROM LEFT TO RIGHT.

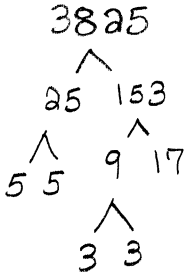
$$\begin{array}{r} 9076 \\ +\ 4689 \\ \hline 13000 \\ 600 \\ 150 \\ 15 \\ \hline 13,765 \end{array}$$

IT IS EQUIVALENT TO

$(500 + 60 + 8) + (700 + 50 + 7)$

$= (500 + 700) + (60 + 50) + (8 + 7)$

20. (3 points) Use a factor tree to determine the prime factorization of 3825.



$3825 = 3^2 \cdot 5^2 \cdot 17$

21. (3 points) Use the standard long division algorithm to compute the quotient and remainder when $57,354$ is divided by 7. Then use the short division algorithm.

$$\begin{array}{r} 8193 \text{ r } 3 \\ 7\overline{)57354} \\ 56 \\ \hline 13 \\ 7 \\ \hline 65 \\ 63 \\ \hline 24 \\ 21 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 8193 \text{ r } 3 \\ 7\overline{)5\,7^{1}3^{6}5^{2}4} \end{array}$$

22. (1 point) Which one of the following is true about prime factorizations?

(a) Some of the factors in a prime factorization may be composite numbers.

(b) The prime factorization of a number is unique.

(c) A prime factorization always contains an even number of factors.

(d) Only odd numbers can appear in a prime factorization.

23. (1 point) Estimate the following sum by rounding each addend to the nearest hundred.

$$550 + 147 + 49 + 1449 + 696$$

$600 + 100 + 0 + 1400 + 700 = 2800$

(a) 2800

(b) 2900

(c) 2850

(d) 3000

24. (1 point) Which one of the following is a legitimate test for divisibility by 18?

(a) An integer is divisible by 18 if and only if it is divisible by 3 and 6.

(b) An integer is divisible by 18 if and only if it is divisible by 2 and 6.

(c) An integer is divisible by 18 if and only if it is divisible by 2 and 9.

(d) An integer is divisible by 18 if and only if the number formed by its last two digits is divisible by 18.

25. (1 point) What is the sign of $-3 \cdot (-2) \div (-6) \cdot (-8) \div (-2) \cdot (-5) \cdot (3)$?

(a) Negative

(b) Positive

26. (1 point) Suppose that $d \mid a$ and $d \mid b$. Which one of the following is NOT necessarily true?

(a) $a \mid b$

(b) $d \mid (a + b)$

(c) $d \mid (a - b)$

(d) $d \mid ab$

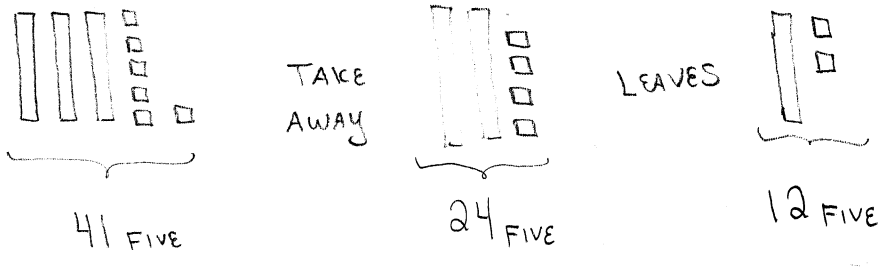
27. (1 point) Compute $41_{\text{five}} - 24_{\text{five}}$.

(a) 17_{five}

(b) 23_{five}

(c) 11_{five}

(d) 12_{five}

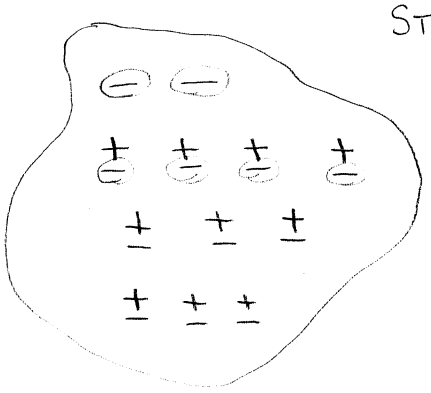


28. (3 points) In each case, use a different integer subtraction model to model and find the difference.

(a) $-2-(-6)$

$= 4$

CHARGED FIELD:

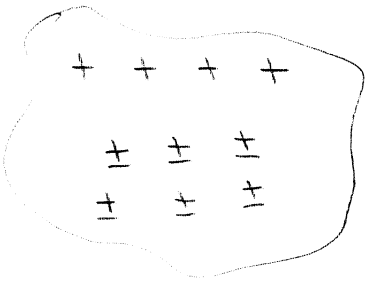


START WITH -2 CHARGE

AND TAKE AWAY

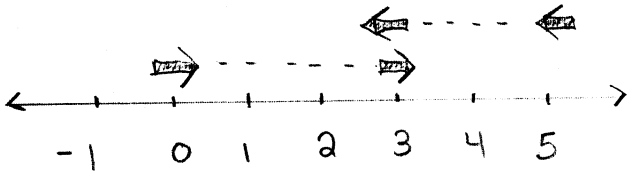
6 NEGATIVES

LEFT WITH $+4$



(b) $3-(-2) = 5$

NUMBER LINE:



START AT ZERO FACING RIGHT.

MOVE FORWARD 3.

TURN AROUND (TO FACE LEFT).

MOVE BACK 2.

END AT 5.