Math 200 - 1st Final Exam

May 5, 2010

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary. Multiple choice problems are worth 0, 1, or 2 points depending on your answer and the work shown.

1. Which one of the following is an example of inductive reasoning?

   (a) 5648 is divisible by 4 because 48 is divisible by 4.

   (b) The next term of the sequence $5, 9, 13, 17, \ldots$ is 21.

   (c) $2(x+4) = 2x+8$

   (d) 17 is a prime number because its only factors are 1 and 17.

   GENERALIZATION BASED ON OBSERVATION

2. Use any appropriate algorithm to compute $34_{\text{six}} \cdot 45_{\text{six}}$.

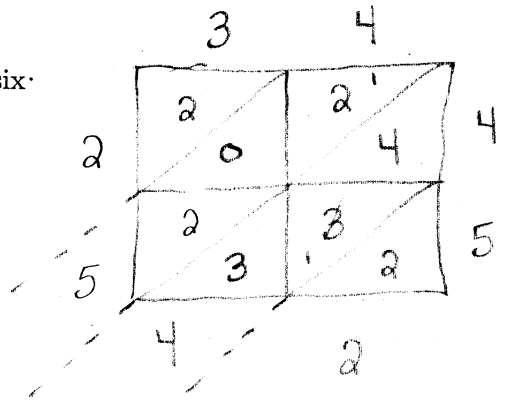
   (a) 2542_{six}

   (b) 2010_{six}

   (c) 1530_{six}

   (d) 2502_{six}

   BASE 6



3. When using the 4-step, problem-solving process which one of these strategies would NOT be considered part of understanding the problem?

   (a) Look for a pattern. ← PART OF DEVISING A PLAN

   (b) State the problem in your own words.

   (c) Read and reread the problem.

   (d) Determine what information is irrelevant.

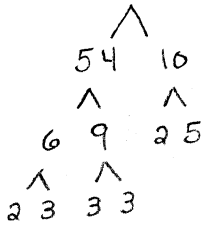
4. How many positive divisors does 540 have?

   (a) 20

   (b) 6

   (c) 18

   (d) 24



$540 = 2^2 \cdot 3^3 \cdot 5$

OF DIVISORS IS

$3 \times 4 \times 2 = 24$

5. Choose the multiplication model that best fits the following problem situation: *A local restaurant offers combo meals with choices of 7 different entrees and 5 different appetizers. How many different combo meals are offered?*

   (a) missing factor

   (b) array

   (c) repeated addition

   (d) Cartesian product

6. Which one of the following facts demonstrates the commutative property of multiplication?

   (a) $2(x+1)+4\cdot 5 = 2(x+1)+5\cdot 4$

   (b) $(6x+10) = 2(3x+5)$

   (c) $2\cdot(6\cdot 7)+1 = (2\cdot 6)\cdot 7+1$

   (d) $(4+1)+2 = 4+(1+2)$

7. Consider the following conjecture:

   If $x \mid y$, then $x \mid (y+2)$.

   Which one of the following is a counterexample?

   (a) $2 \mid 6$ and $2 \mid (6+2)$.

   (b) $5 \mid 10$ and $5 \nmid (10+2)$

   (c) $3 \nmid 4$ and $3 \mid (4+2)$

   (d) The conjecture is true.

8. When using the 4-step problem-solving process, which one of these strategies would NOT be considered part of looking back?

   (a) Generalize.

   (b) Consider other possible solution methods.

   (c) Keep notes of your work.

   (d) Check your answer in the original wording of the problem.

9. Rewrite 6530_{seven} in base ten.

   (a) 25,106

   (b) 2,324

   (c) 16,268

   (d) 4,571

$6\times 7^3 + 5\times 7^2 + 3\times 7^1 + 0\times 7^0$

10. A sequence is defined recursively as follows:

$$a_1 = 7; \quad a_n = 3 \cdot a_{n-1} - 11, \text{ for } n = 2, 3, 4, \ldots$$

Find a_3 (the third term of the sequence).

$a_1 = 7$

$a_2 = 3 \cdot a_1 - 11 = 21 - 11 = 10$

$a_3 = 3 \cdot a_2 - 11 = 30 - 11 = 19$

(a) 19
(b) 18
(c) 10
(d) -5

11. Which one of the following divisibility tests is incorrect?

(a) A whole number is divisible by 3 if the sum of its digits is divisible by 3.
(b) A whole number is divisible by 6 if is divisible by both 2 and 3.
(c) A whole number is divisible by 9 if the number formed by its last 2 digits is divisible by 9.
(d) A whole number is divisible by 8 if the number formed by its last three digits is divisible by 8.

12. Which one of these numbers is the 7th term of the following sequence?

3, 8, 15, 24, 35, ... 48 63

5 7 9 11 13 15

(a) 45
(b) 63
(c) 48
(d) 59

13. (5 points) John raises llamas and ostriches. Looking out onto his pasture, he counts 10 heads and 28 legs. How many of each type of animal is on John's pasture? (Use any technique to solve the problem, but show your work and check your answer. You will receive no credit for simply writing a answer.)

Let l = # of Llamas

o = # of ostriches

$l + o = 10 \rightarrow o = 10 - l$

$4l + 2o = 28$

$4l + 2(10 - l) = 28$

$4l + 20 - 2l = 28$

$2l = 8$

$l = 4$

$o = 6$

Check:

Heads: $4 + 6 = 10$ ✓

Legs: $4(4) + 2(6) = 16 + 12 = 28$ ✓

14. (1 point) Referring to the 4-step, problem-solving process, state two different strategies for carrying out the plan.

* Implement the plan devised in Step 2 of the process
* Check your work at each step as you proceed
* Keep notes of your work

15. (5 points) Write a word problem involving whole number division that is most naturally modeled using each of the following models. Write a different problem for each model and write the corresponding division fact.

(a) Partition: Sam has 24 Pokemon cards that he will divide evenly among 4 friends. How many cards does each friend get?

$24 \div 4 = 6$

(b) Repeated subtraction:

Sam has 36 darts for his Nerf dart gun. He will give 6 darts to each of his friends until he has none left. How many friends get darts?

$36 \div 6 = 6$

16. (5 points) List all prime numbers less than 30. Briefly explain how your list of primes is big enough to determine whether 883 is prime. Is 883 prime?

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

To determine if 883 is prime, we must check for divisibility by all primes up to $\sqrt{883} \approx 29.715$

883 is not divisible by any number in our list. It is prime.

17. (5 points) Clearly state the rule for adding two integers with opposite signs. Give an example that illustrates your rule.

SUBTRACT THE ABSOLUTE VALUES, LEAST FROM GREATEST. THEN GIVE YOUR RESULT THE SIGN OF THE ORIGINAL ADDEND WITH THE GREATEST ABSOLUTE VALUE.

Ex

$4 + (-9) = -(9-4) = -5$

18. (5 points) What does it mean for a number to be prime? Use what you've said to explain why 1 is not a prime number. Use what you've said to explain why 2 is a prime number.

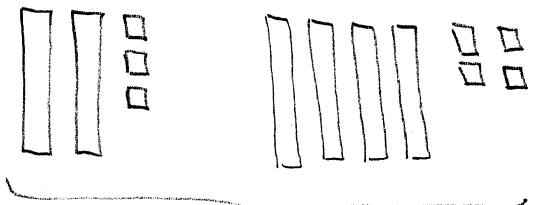
A POSITIVE INTEGER IS PRIME IF IT HAS EXACTLY TWO DIFFERENT POSITIVE INTEGER FACTORS.

1 IS NOT PRIME BECAUSE IT HAS ONLY ONE FACTOR, ITSELF.

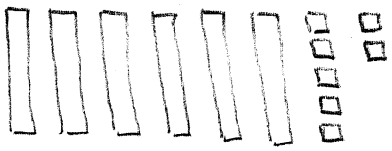
2 IS PRIME BECAUSE ITS ONLY FACTORS ARE 1 & 2.

19. (5 points) Compute the sum of 23_{five} and 44_{five}. Use base-five blocks to illustrate your procedure.

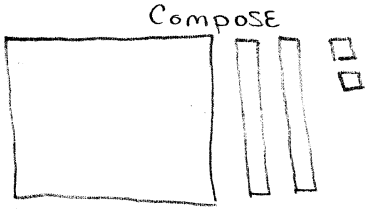
$$\begin{array}{r} {\scriptstyle 1\ \ \ } \\ 23_{\text{FIVE}} \\ +\ 44_{\text{FIVE}} \\ \hline 122 \end{array}$$



COMBINE



COMPOSE



20. (5 points) Use the Euclidean algorithm to compute the GCD and LCM of 12920 and 419.

$$419\overline{)12920}\quad 30,\ 12570,\ 350 \qquad 350\overline{)419}\quad 1,\ 350,\ 69 \qquad 69\overline{)350}\quad 5,\ 345,\ 5$$

GCD(419, 350) GCD(350, 69) GCD(69, 5) = 1

GCD = 1

$\Rightarrow$ LCM = 419 × 12920 = 5,413,480

21. (5 points) Marijke used the number line to model $-3 - 5$. Here is what she said:

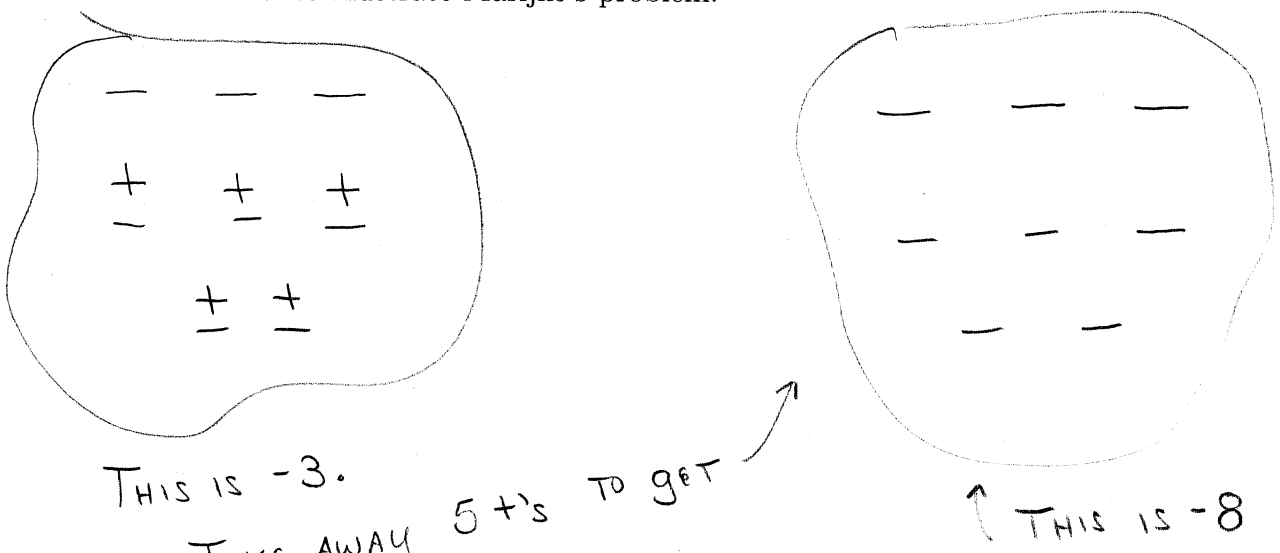
"Start at zero, facing right. Turn around, go three. Turn around, go five. You end up at two. Therefore $-3 - 5 = 2$."

(a) Is Marijke correct? If not, correct her application of the number line model.

No way. $-3-5=-8$

" Start at zero, facing right. Back up 3. Turn around, go five. You end up at -8."

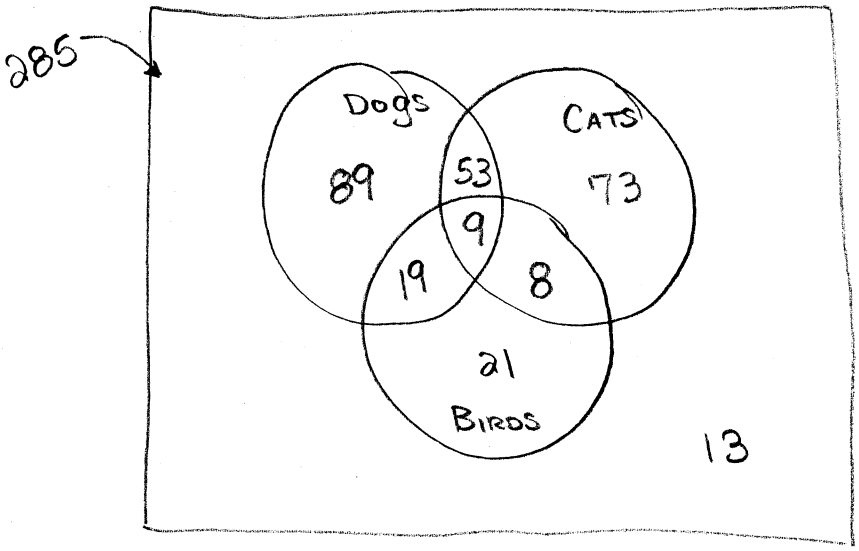
(b) Use a different model to illustrate Marijke's problem.



22. (5 points) In a survey of 285 pet lovers, the following information was obtained:

- 170 had pet dogs;
- 143 had pet cats;
- 57 had pet birds;
- 62 had both dogs and cats;
- 28 had both dogs and birds;
- 17 had both cats and birds; and
- 9 had all three types of pets.

Organize this data in a three-set Venn diagram. How many people surveyed had none of these types of pets?



89 + 53 + 73 + 19 + 9 + 8 + 21 = 272

285 - 272 = 13

23. (5 points) Let A be the set of all natural numbers, let B be the set of all integers, and let $C = \{-2, -3, 2, 5\}$.

(a) List the elements of $A \cap C$. $\{2, 5\}$

(b) Which set is bigger $\underbrace{A \cap B}_{A}$ or $\underbrace{A \cup B}_{B}$? $A \cup B$ is bigger

Both sets are infinite, but $(A \cap B) \subset (A \cup B)$.

(c) List two elements of $C \times B$.

$(-2, 1)$, $(5, 8)$

(d) What is the value of $n(C)$?

4

(e) List the elements of $C - A$.

$\{-2, -3\}$