

Math 200 - 2nd Final Exam
 May 17, 2010

Name key
 Score _____

Show all work to receive full credit. Supply explanations where necessary. Multiple choice problems are worth 0, 1, or 2 points depending on your answer and the work shown.

1. When using the 4-step, problem-solving process which one of these strategies would NOT be considered part of devising a plan?

- (a) Look for a pattern.
- (b) Sketch a picture.
- (c) Examine a smaller problem.
- (d) Determine what information is irrelevant. PART OF UNDERSTAND THE PROBLEM

2. Choose the subtraction model that best fits the following problem situation: *Caleb has 5 cookies and Katie has 3 cookies. How many more cookies does Caleb have?*

- (a) comparison
- (b) take away
- (c) number line
- (d) missing addend

3. Use any appropriate addition algorithm to compute the following sum.

$$33_{\text{four}} + 22_{\text{four}} + 32_{\text{four}} + 1_{\text{four}} + 20_{\text{four}} + 3_{\text{four}} + 13_{\text{four}}$$

- (a) 124_{four}
- (b) 130_{four}
- (c) 322_{four}
- (d) 333_{four}

3	3
2	2
3	2
2	X
2	0
1	3
3	2
2	2

4. Which one of the following facts demonstrates the associative property of addition?

- (a) $2(5 + 1) + 3 = 2(1 + 5) + 3$
- (b) $(6x + 10) = 2(3x + 5)$
- (c) $(x + 3) + (2 + y) = (y + 2) + (x + 3)$
- (d) $(4 + 1) + 2 = 4 + (1 + 2)$

5. Which one of the following is not a basic property of the Hindu-Arabic numeration system?
- (a) Place value is based on powers of 10.
 - (b) Each digit in a numeral has a face value, a place value, and a value.
 - (c) Each number is either prime or composite.
 - (d) All numerals are constructed from the 10 digits.

6. Consider the following conjecture:

$$\text{If } 3 \mid x, \text{ then } 6 \mid x.$$

Which one of the following is a counterexample?

- (a) $3 \mid 18$ and $6 \mid 18$.
 - (b) $3 \nmid 10$ and $6 \nmid 10$
 - (c) $3 \mid 27$ and $6 \nmid 27$
 - (d) The conjecture is true.
7. Which one of these fractions is in lowest terms?

- (a) $30/147 = 10/49$
- (b) $5/40 = 1/8$
- (c) $72/35$
- (d) $51/85 = 3/5$

8. Which one of these numbers is the 835th term of the following arithmetic sequence?

$$10, 13, 16, 19, 22, 25, \dots$$

- (a) 2512
 - (b) 2518
 - (c) 2515
 - (d) 2524
- $$N^{\text{TH}} = 3N + 7$$
- $$835^{\text{TH}} = 3(835) + 7 =$$

9. Which one of the following is an example of deductive reasoning?

- (a) Every elephant I have ever seen is gray. Therefore every elephant must be gray.
- (b) It has rained every day. Therefore it will rain tomorrow.
- (c) Since multiplication is commutative, $15 \cdot 37$ must be equal to $37 \cdot 15$.
- (d) A sequence begins with 1,2,3,4. The next term must be 5.

10. Choose the division model that best fits the following problem situation: 52 playing cards are shared equally among 4 people. How many cards does each person get?

- (a) partition
- (b) missing factor
- (c) repeated subtraction
- (d) division algorithm

11. Let $A = \{0, 1, 2, 3\}$. Choose the set B for which all of the following are true?

$$4 \in B, \quad A \sim B, \quad A \cap B = \emptyset, \quad (7, 2) \in B \times A$$

- (a) $B = \{4, 7\}$
- (b) $B = \{4, 5, 7, 8\}$
- (c) $B = \{3.5, 4, 8, 9\}$
- (d) $B = \{0, 2, 4, 7\}$

12. Convert 4103_{six} to base ten.

- (a) 5403
- (b) 903
- (c) 5418
- (d) 3555

$$4 \times 6^3 + 1 \times 6^2 + 0 \times 6 + 3 =$$

13. (5 points) In the harbor there were ships with 3 masts and ships with 4 masts. John saw 7 ships and counted a total of 26 masts. How many ships had 3 masts and how many had 4? (Use any technique to solve the problem, but show your work. You will receive no credit for simply writing an answer.)

LET $x = \#$ OF SHIPS WITH 3 MASTS

LET $y = \#$ OF SHIPS WITH 4 MASTS

$$\begin{aligned} x + y &= 7 \\ 3x + 4y &= 26 \end{aligned} \Rightarrow \begin{aligned} -3x - 3y &= -21 \\ \underline{3x + 4y} &= 26 \end{aligned}$$

$$y = 5$$

$$x = 2$$

CHECK:

$$\text{SHIPS: } 2 + 5 = 7 \checkmark$$

$$\text{MASTS: } 3(2) + 4(5) = 6 + 20 = 26 \checkmark$$

2 SHIPS WITH 3 MASTS
5 SHIPS WITH 4 MASTS

14. (1 point) Give an example of a geometric sequence.

1, 2, 4, 8, 16, 32, ...

First Term is 1

Ratio is 2

15. (5 points) Clearly state the rule for adding two integers with opposite signs. Give an example that illustrates your rule.

SUBTRACT THE ABSOLUTE VALUES, LEAST FROM GREATEST.

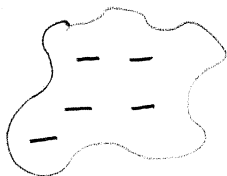
GIVE YOUR RESULT THE SIGN OF THE ORIGINAL ADDEND WITH THE GREATEST ABSOLUTE VALUE.

Ex $3 + (-7) = -(7 - 3) = -4$

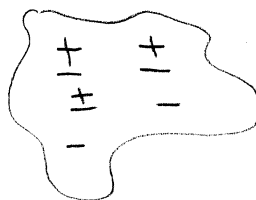
16. (5 points) Use a model to illustrate each of the following. (Model what is given, not a related problem.)

(a) $-5 + 3$

START WITH -5



PUT IN 3

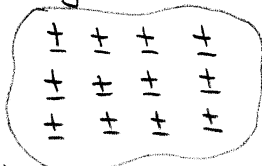


\pm CANCEL TO LEAVE

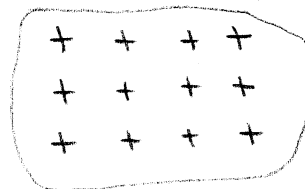
-2

(b) $(-3) \times (-4)$

START WITH FIELD OF CHARGE ZERO.



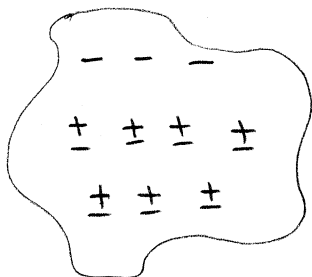
THEN TAKE OUT 3 groups OF 4 NEGATIVES.



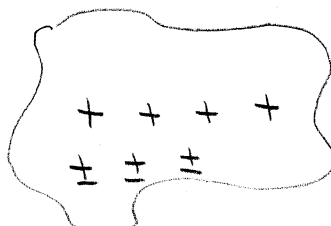
LEAVES +12

(c) $-3 - (-7)$

START WITH -3

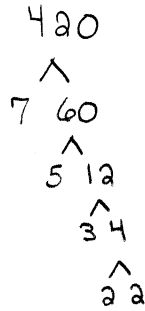
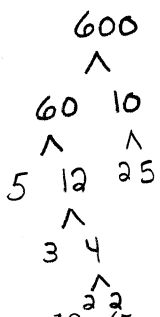


THEN TAKE AWAY 7 NEGATIVES



LEAVES +4

17. (5 points) Find the prime factorizations of 600 and 420. Then find the GCD and LCM.



$$\begin{array}{l}
 600 = 2^3 \cdot 3 \cdot 5^2 \\
 420 = 2^2 \cdot 3 \cdot 5 \cdot 7
 \end{array}$$

$$\begin{array}{l}
 \text{GCD} = 2^2 \cdot 3 \cdot 5 = 60 \\
 \text{LCM} = 2^3 \cdot 3 \cdot 5^2 \cdot 7 \\
 = 4200
 \end{array}$$

18. (5 points) Let $A = \{1, 2, 3\}$ and $B = \{x, y\}$.

(a) Find the Cartesian product $A \times B$.

$$A \times B = \{ (1, x), (2, x), (3, x), (1, y), (2, y), (3, y) \}$$

(b) Write a word problem involving the multiplication fact $3 \times 2 = 6$ that is most naturally modeled using the Cartesian product model.

JOE HAS 3 SHIRTS AND 2 SHORTS. HOW MANY SHIRT / SHORT COMBINATIONS DOES HE HAVE ?

19. (5 points) Classify each of the following as true or false. If false, provide a counterexample.

(a) If $n(A) = n(B)$, then $A = B$.

FALSE -- $A = \{1\}$, $B = \{2\}$ EACH HAS 1 ELEMENT, BUT $A \neq B$.

(b) If $A - B = \emptyset$, then $A = B$.

FALSE -- $A = \{a, b\}$, $B = \{a, b, c\}$
 $A - B = \emptyset$ BUT $A \neq B$.

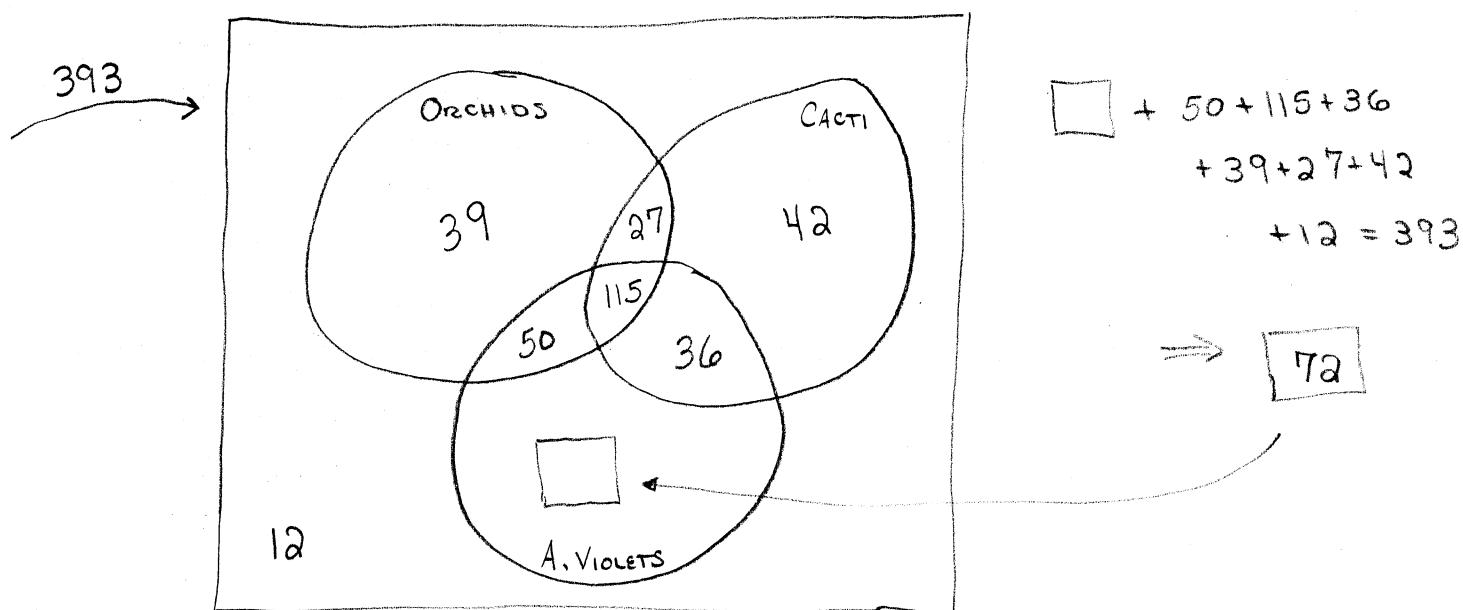
(c) If $A \subseteq B$, then $A \cap B = A$.

TRUE

20. (5 points) In a survey of 393 horticulturists, the following information was obtained:

- 231 keep orchids;
- 220 keep cacti;
- 165 keep African violets and orchids;
- 151 keep African violets and cacti;
- 142 keep orchids and cacti;
- 115 keep all three types of plants; and
- 12 keep none of these types of plants.

Organize the data in a three-set Venn diagram. How many people surveyed keep only African violets?



21. (5 points) Give an example of a 7-digit number that is divisible by 2, 3, and 4, but not by 8, 9, and 10. Explain how you know.

1,011,036

By 2 BECAUSE $2 \mid 6$

By 3 BECAUSE $(1+1+1+3+6) = 12$ AND $3 \mid 12$

By 4 BECAUSE $4 \mid 36$

Not by 8 BECAUSE $8 \nmid 36$

Not by 9 BECAUSE $1+1+1+3+6 = 12$ AND $9 \nmid 12$

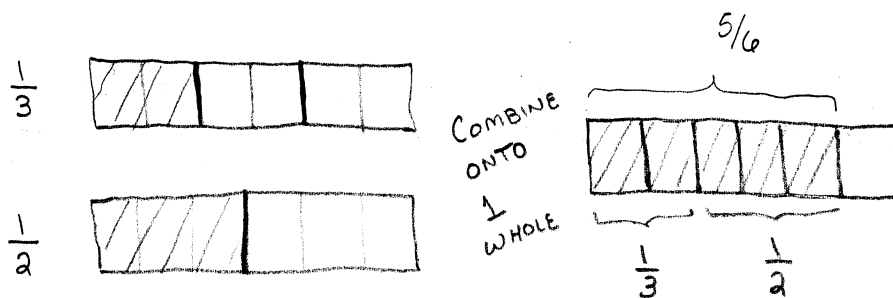
Not by 10 BECAUSE DOESN'T END WITH 0.

22. (5 points) State the four steps of the 4-step, problem-solving process. Choose any one of your steps (Tell which you choose!) and list two strategies associated with that step.

- ① UNDERSTAND THE PROBLEM
- ② DEVISE A PLAN
- ③ CARRY OUT THE PLAN
- ④ LOOK BACK
 - (i) CHECK
 - (ii) GENERALIZE

23. (5 points) Compute each of the following. Then sketch a figure and use portions to illustrate each one.

(a) $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$



(b) $\frac{3}{4} \cdot \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$

