

Math 200 - Test 1
February 15, 2012

Name key Score _____

Show all work to receive full credit (even on multiple-choice problems). Supply explanations when necessary.

1. (4 points) Clearly state the four steps of the problem-solving process (in order).

- ① UNDERSTAND THE PROBLEM ③ CARRY OUT THE PLAN
② DEVISE A PLAN ④ LOOK BACK

2. (5 points) Each of the following strategies is associated with one of the four steps of the problem-solving process. Write the number of the step (1, 2, 3, or 4) that corresponds to each strategy.

- (a) Check your work at each step. 3
(b) Make a table, list, or diagram. 2
(c) Generalize. 4
(d) Identify a subgoal. 2
(e) State the problem in your own words. 1
(f) What information is missing or not needed? 1
(g) Check your solution in the original wording of the problem. 4
(h) Write an equation. 2
(i) Keep an accurate record of your work. 3
(j) Look for a pattern. 2

3. (1 point) What is often used to disprove a conjecture?
- (a) Circular reasoning
 - (b) Working backwards
 - (c) A counterexample
 - (d) Inductive reasoning
4. (1 point) What kind of sequence of numbers is associated with consecutive terms having a common ratio?
- (a) A figurate sequence
 - (b) A geometric sequence
 - (c) An arithmetic sequence
 - (d) A Bobo sequence
5. (1 point) Which one of the following sets is NOT empty?
- (a) { }
 - (b) The set of all natural numbers less than 1
 - (c) \emptyset
 - (d) $\{\emptyset\}$ ← HAS ONE ELEMENT IN IT.
6. (1 point) Which of the following is the 5th term of the arithmetic sequence whose 1st term is 7 and whose difference is 9?
- (a) 7^8
 - (b) 52
 - (c) 45
 - (d) 43
- $7, 16, 25, 34, 43, 52, \dots$
7. (1 point) Which one of the statements below would be read "x is an element of A?"
- (a) $x \subseteq A$
 - (b) $x \in A$
 - (c) $x \sim A$
 - (d) $\{x | x \in A\}$

8. (2 points) Consider the following statement: If a word starts with the letter c , then the c makes the sound of an s . Give an example and a counterexample. Be sure to state which is which.

EXAMPLE :

CITY
(C MAKES SOUND
OF S)

COUNTEREXAMPLE :

CAR
(C DOES NOT MAKE
SOUND OF S)

9. (2 points) Suppose U is the set of all U.S. presidents, and A is the subset of living presidents. What are the elements of \bar{A} ? (Describe the elements in words.)

\bar{A} = SET OF ALL DECEASED U.S. PRESIDENTS
= { WASHINGTON, ADAMS, ..., REAGAN }

10. (3 points) Find the 917th term of the arithmetic sequence shown below.

17, 21, 25, 29, 33, ...

$\begin{matrix} \vee & \vee \\ 4 & 4 \end{matrix}$

$$N^{\text{TH}} \text{ TERM} = 4N + 13$$

$$917^{\text{TH}} \text{ TERM} = 4(917) + 13$$

$$= \boxed{3681}$$

11. (3 points) A compass and ruler together cost \$4. The compass costs 90 cents more than the ruler. How much do three compasses cost? (You must show work to receive full credit.)

LET C = COST OF COMPASS

r = COST OF RULER

$$C = r + 0.90$$

$$C + r = 4$$

SUBSTITUTE $C = r + 0.90$

$$(r + 0.90) + r = 4$$

$$2r + 0.90 = 4$$

$$2r = 3.10$$

$$r = 1.55$$

$$C = 1.55 + 0.90 = 2.45$$

THREE COMPASSES COST

$$3 \times \$2.45 = \boxed{\$7.35}$$

12. (3 points) Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8, 10\}$, and $C = \{1, 3, 5\}$. Determine each of the following.

(a) $A \cap B = \{2, 4\}$

(b) $B - A = \{6, 8, 10\}$

(c) $n(C) = 3$

(d) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 10\}$

(e) $B \cap C = \{\} = \phi$

(f) $\emptyset \cup C = C = \{1, 3, 5\}$

13. (2 points) Give an example of a geometric sequence and write a formula for the n th term of your sequence.

$$3, 6, 12, 24, 48, 96, \dots$$

$$N^{\text{TH}} \text{ Term} = 3 \cdot 2^{N-1}$$

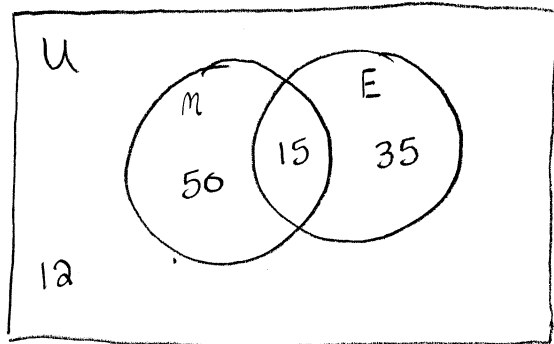
14. (3 points) The first difference of a sequence is 2, 4, 6, 8, 10, The fifth term of the original sequence is 35. Find the first six terms of the original sequence.

Original Sequence →	<u>15</u>	,	<u>17</u>	,	<u>21</u>	,	<u>27</u>	,	<u>35</u>	,	<u>45</u>
	√		√		√		√		√		
	2		4		6		8		10		
			4								

15. (4 points) A group of PSC students were surveyed and the following information was obtained:

- 65 students were taking a math class
- 50 students were taking an English class
- 15 students were taking both an English class and a math class
- 12 students were taking neither an English class nor a math class

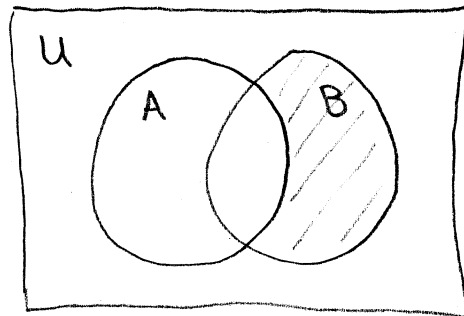
Sketch a two-set Venn diagram displaying the information given above. Use your Venn diagram to determine how many students were surveyed.



$$50 + 15 + 35 + 12 = 112$$

112 were surveyed.

16. (2 points) Sketch a two-set Venn diagram and shade the region corresponding to $\bar{A} \cap B$.



↑
OUTSIDE A AND IN B

17. (3 points) Rewrite each of the following statements using mathematical symbols.

(a) The set D is equivalent to the set E .

$$D \sim E$$

(b) The cardinality of the union of the two sets P and Q is 13.

$$N(P \cup Q) = 13$$

(c) The set F is a subset of the empty set.

$$F \subseteq \emptyset$$

18. (3 points) Use the technique we discussed in class to compute the sum:

$$S = 2 + 4 + 6 + 8 \dots + 422 + 424. \quad \leftarrow 212 \text{ TERMS}$$

$$S = \underline{424 + 422 + \dots + 4 + 2}$$

$$2S = 426 \times 212$$

$$S = \frac{426 \times 212}{2} = \boxed{45,156}$$

19. (2 points) Give an example of a "set" that is not well defined, and explain why your set is not well defined.

THE SET OF ALL PRETTY FLOWERS IS NOT WELL DEFINED BECAUSE IT IS NOT CLEAR WHAT FLOWERS ARE PRETTY FLOWERS.

20. (3 points) Determine all of the subsets of $\{1, 2, 3\}$.

$$\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$$

8 SUBSETS IN ALL.

21. (1 point) Rewrite the set S in roster (listing) notation.

$$S = \{x \mid x \in \mathbb{N} \text{ and } 100 \leq x < 107\}$$

$$= \{100, 101, 102, 103, 104, 105, 106\}$$