# Math 200 - Test 2

March 14, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (1 point) Which property justifies the following fact: $(x+y)+7 = 7+(x+y)$?

   (a) Associative property of addition

   (b) Commutative property of addition

   (c) Closure property of whole-number addition

   (d) Distributive property of multiplication over addition

2. (2 points) Use any addition algorithm to compute $323_{\text{four}} + 133_{\text{four}} + 322_{\text{four}} + 313_{\text{four}}$.

   (a) $1091_{\text{four}}$

   (b) $1121_{\text{four}}$

   (c) $3122_{\text{four}}$

   (d) $3023_{\text{four}}$

$$\begin{array}{r} {\scriptstyle 2\ 2} \\ 3\ 2\ 3 \\ 1\ 3\ 3 \\ 3\ 2\ 2 \\ +\ 3\ 1\ 3 \\ \hline 3\ 0\ 2\ 3 \end{array}$$

3. (1 point) Choose the subtraction model that best fits the following problem situation: *Al has read 4 chapters of a 9-chapter book. How many chapters does he have left to read?*

   (a) comparison model

   (b) take-away model

   (c) set partition model

   (d) missing addend model

4. (1 point) Suppose $A$ and $B$ are sets with $n(A) = 12$ and $n(B) = 4$. Determine $n(B \times A)$.

   (a) 16

   (b) 48

   (c) $(4, 12)$

   (d) $(12, 4)$

$n(B \times A) = 4 \times 12 = 48$

5. (1 point) Choose the multiplication model that best fits the following problem situation: *The local sub shop offers 7 different bread choices and 9 different meat choices. How many different bread-meat combinations are possible?*

   (a) Cartesian product model

   (b) set partition model

   (c) area/array model

   (d) repeated addition model

6. (1 point) What is the value of the digit 7 in the numeral $6782_{\text{nine}}$?

(a) 567
(b) 7
(c) 700
(d) 81

$7 \times 9^2 = 7 \times 81 = 567$

7. (1 point) What base-six number immediately follows $5355_{\text{six}}$?

(a) $5356_{\text{six}}$
(b) $5360_{\text{six}}$
(c) $5500_{\text{six}}$
(d) $5400_{\text{six}}$

8. (1 point) Choose the addition model that best fits the following problem situation: *Marie has 7 pieces of yarn in one pile and 4 pieces of yarn in another pile. If she combines the piles, how many pieces of yarn will she have in the combined pile?*

(a) group partition model
(b) set model
(c) Cartesian sum model
(d) number line model

9. (1 point) Which one of these illustrates the associative property of multiplication?

(a) $(7 \times y) \times 3 = 3 \times (7 \times y)$
(b) $3(x + 5) = 3x + 15$
(c) $(3 + z) + 7 = 3 + (z + 7)$
(d) $5(xy) = (5x)y$

10. (1 point) What algorithm is being illustrated here?

$$\begin{array}{r} 72 \\ -\,38 \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 72 \;+\; 2 \\ -\,(38 \;+\; 2) \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 74 \\ -\,40 \\ \hline 34 \end{array}$$

(a) scratch subtraction algorithm
(b) fast subtraction algorithm
(c) equal-additions algorithm
(d) nice-numbers algorithm

11. (3 points) Write a word problem involving multiplication in which the multiplication fact is best described by the array model.

THE COOKIES ON THE COOKING SHEET WERE ARRANGED INTO A RECTANGULAR ARRAY THAT HAD 4 ROWS AND 3 COOKIES IN EACH ROW. HOW MANY COOKIES WERE ON THE SHEET?

12. (3 points) Use any of the algorithms we discussed in class, except the standard algorithm, to compute $2796 + 3458$.

LATTICE:

$$\begin{array}{r} 2796 \\ +\ 3458 \end{array}$$

| | | | |
|---|---|---|---|
| 0 | 1 | 1 | 1 |
| 5 | 1 | 4 | 4 |

$6254$

PARTIAL SUMS:

$$\begin{array}{r} 2796 \\ +\ 3458 \\ \hline 14 \\ 140 \\ 1100 \\ 5000 \\ \hline 6254 \end{array}$$

13. (4 points) Let $Y = \{1, 2, 3\}$ and $Z = \{a, b\}$.

(a) Determine $\emptyset \times Z$.

$\emptyset \times Z = \emptyset$

(b) Determine $Z \times Y$.

$\{(a,1), (b,1), (a,2), (b,2), (a,3), (b,3)\}$

(c) What is the difference between $Y \times Z$ and $Z \times Y$?

EACH ELEMENT OF $Y \times Z$ IS THE REVERSE OF THE CORRESPONDING ELEMENT OF $Z \times Y$.

$Y \times Z = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

14. (3 points) Use the abstract version of the set model to illustrate $5 + 3 = 8$.

$A = \{v, w, x, y, z\}$ $\quad n(A) = 5$

$B = \{p, d, q\}$ $\quad n(B) = 3$

$A \cup B = \{p, d, q, w, x, y, z\}$

$5 + 3 = n(A) + n(B)$

$= n(A \cup B) = 8$

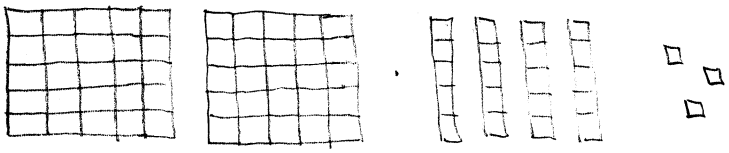
See page 118 and 119.

15. (3 points) Use any **two** of the strategies for mastering basic addition facts to compute $8+6$. To receive any credit, you must show work and explain your reasoning.

Doubles: Take from one to make doubles. Then add what's left over. $8+6 = 2+(6+6) = 2+12 = 14$

Making 10: Take from one to make ten. Then add what's left over. $8+6 = (8+2)+4 = 10+4 = 14$

16. (3 points) Use base-five blocks to represent $243_{\text{five}}$. Then use your representation to determine the numbers's value in base ten.



2 flats 4 longs 3 units

$243_{\text{five}} = 2\times 25 + 4\times 5 + 3 = 73$

17. (3 points) Use any of the algorithms we discussed in class, except the standard algorithm, to compute $573 - 287$.

Equal-additions:

$$\begin{array}{r} 573 \\ -\ 287 \\ \hline \end{array} \xrightarrow[3]{\text{add}} \begin{array}{r} 576 \\ -\ 290 \\ \hline \end{array} \xrightarrow[10]{\text{add}} \begin{array}{r} 586 \\ -\ 300 \\ \hline 286 \end{array}$$

Left-to-right:

$$\begin{array}{r} {\scriptstyle 4}\ {\scriptstyle 16}\ {\scriptstyle 13} \\ 5\not{7}\not{3} \\ -\ 287 \\ \hline 286 \end{array}$$

18. (3 points) Give an example of a set that is closed under addition and an example of a set that is not closed under addition. Give a brief explanation for each set.

$\{0, 2, 4, 6, 8, \ldots\}$ Closed because the sum of two evens is an even.

$\{1, 3, 5, 7, \ldots\}$ Not closed because $1+3=4$ and 4 is not in the set.

19. (3 points) Convert 237 to base three.

$3^0 = 1,\ 3^1 = 3,\ 3^2 = 9,\ 3^3 = 27,\ 3^4 = 81,\ 3^5 = 243$

$3^4 = 81 \overline{)237}\ 2$ (-162)

$3^3 = 27 \overline{)75}\ 2$ (-54)

$3^2 = 9 \overline{)21}\ 2$ (-18)

$3^1 = 3 \overline{)3}\ 1$ (-3)

$3^0 = 1 \overline{)0}\ 0$

$237 = 22210_{\text{THREE}}$

20. (1 point) State one of the important properties of the Hindu-Arabic numeration system.

See page 63.

① All numerals are constructed from the ten basic digits

② Place value is based on powers of 10

21. (2 points) Rewrite each expression using the indicated property, and only that property, exactly one time.

(a) *Commutative property of multiplication:* $y + 8(3 + w) = y + (3+w)8$

(b) *Associative property of multiplication:* $5[3(x + 2)] = (5 \cdot 3)(x + 2)$

22. (2 points) List the first six natural numbers in base two.

1, 10, 11, 100, 101, 110, ...

All in base 2

23. (3 points) Use a multiplcation model to illustrate the following fact.

$$3 \cdot (w+2) = 3 \cdot w + 3 \cdot 2$$

REPEATED ADDITION

$3 \cdot (w+2) = (w+2) + (w+2) + (w+2)$

$= (w + w + w) + (2 + 2 + 2)$

$= 3 \cdot w + 3 \cdot 2$

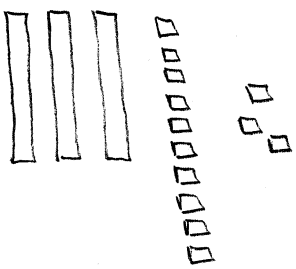
24. (3 points) Use base-ten blocks to illustrate $43 - 28$.

← ILLUSTRATES 43, i.e 4 LONGS & 3 UNITS

WE WANT TO REMOVE 28, i.e. 2 LONGS AND 8 UNITS.

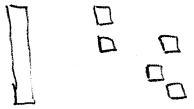
WE NEED TO DECOMPOSE A LONG INTO 10 UNITS.

43 = 3 LONGS AND 13 UNITS



NOW REMOVE 2 LONGS AND 8 UNITS.

LEFT WITH



WHICH REPRESENTS 15.