Math 200 - Test 3
April 24, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) Write an application problem (i.e. a word problem) involving the division fact $12 \div 4 = 3$ where the division is best represented by the following model.

   (a) Repeated subtraction

   TWELVE TOMATO PLANTS ARE PLANTED IN GROUPS OF FOUR PLANTS EACH. HOW MANY GROUPS OF TOMATO PLANTS ARE THERE?

   (b) Set (partition)

   TWELVE COOKIES ARE DIVIDED EVENLY INTO FOUR GROUPS. HOW MANY COOKIES ARE IN EACH GROUP?

2. (5 points) What is the difference between $5 \div 0$ and $0 \div 5$?. Which is defined? Why? Which is not defined? Why not? Explain clearly and carefully.

   $0 \div 5$ IS DEFINED. $0 \div 5$ IS THE UNIQUE NUMBER c SUCH THAT $5 \cdot c = 0$. THAT UNIQUE NUMBER c IS 0. $0 \div 5 = 0$ BECAUSE $5 \cdot 0 = 0$.

   $5 \div 0$ IS NOT DEFINED. THERE IS NO SUCH NUMBER c SUCH THAT $c \cdot 0 = 5$, THEREFORE $5 \div 0$ CANNOT BE DEFINED.

3. (6 points) Use any algorithm except the standard algorithm to compute each product. Use a different algorithm for each part.

(a) 678×54

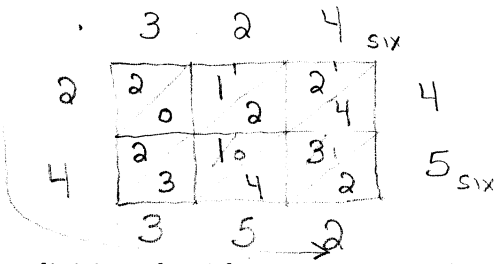
PARTIAL PRODUCTS

$$\begin{array}{r} 678 \\ \times\ 54 \\ \hline 32 \\ 280 \\ 2400 \\ 400 \\ 3500 \\ 30000 \\ \hline 36612 \end{array}$$

$678 \times 54 = 36,612$

(b) $45_{six} \times 324_{six}$

LATTICE



$45_{SIX} \times 324_{SIX} = 24352_{SIX}$

4. (2 points) Use the short division algorithm to compute $8739 \div 7$. Be sure that I can follow your use of this particular algorithm.

$$7\overline{)8739} \quad 1248 \text{ r } 3$$

5. (2 points) Use one of the computational estimation techniques of section 3.5 to estimate the following sum. Briefly describe your strategy.

$$6200 + 5842 + 6512 + 5512 + 6319$$

USING THE "CLUSTERING" STRATEGY...

EACH NUMBER IS APPROXIMATELY 6000.

THEREFORE, THE SUM IS APPROXIMATELY 6000×5.

$= 30,000.$

6. (7 points) Use a **different** model to model each of the following.

(a) $3+(-5)$

PATTERN

$3+4=7$
$3+3=6$
$3+2=5$
$3+1=4$
$3+0=3$

DECREASING AN ADDEND BY 1, DECREASES THE SUM BY 1

CONTINUING ...

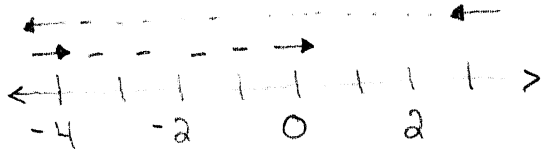
$3+(-1)=2$
$3+(-2)=1$
$3+(-3)=0$
$3+(-4)=-1$
$3+(-5)=2$

(b) $-4-(-7)$

NUMBER LINE

START AT 0 FACING RIGHT
MOVE BACK 4 UNITS
TURN AROUND
MOVE BACK 7 UNITS
END AT +3

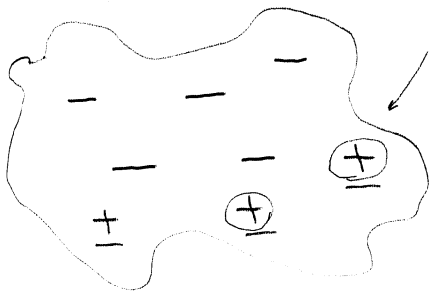
$-4-(-7)=3$



(c) $-5-2$

CHARGES

START WITH 5 NEGS AND REMOVE 2 POS



$-5-2=-7$

7. (3 points) Clearly state the rule for adding two integers with opposite signs. Give an example that illustrates your rule.

TO ADD TWO INTEGERS WITH OPPOSITE SIGNS, SUBTRACT THEIR ABSOLUTE VALUES, LEAST FROM GREATEST. THEN GIVE YOUR RESULT THE SIGN OF THE ORIGINAL INTEGER WITH THE GREATEST ABSOLUTE VALUE.

eg. $5+(-7)=-(7-5)=-2$

8. (3 points) Abigail thought that $-36 \div (-4)$ should be a negative number, because "that's what makes sense." Use the missing-factor model to help Abigail with her problem.

$-36 \div (-4)$ MUST BE A POSITIVE NUMBER, BECAUSE TO DETERMINE $-36 \div (-4)$ WE ASK "WHAT TIMES -4 IS -36?" A POSITIVE TIMES A NEGATIVE IS A NEGATIVE. $\Rightarrow$ ANS MUST BE POSITIVE.

9. (6 points) Use a **different** multiplication model to model product.

(a) $3 \times (-5)$

PATTERN

$3 \times 3 = 9$
$3 \times 2 = 6$
$3 \times 1 = 3$
$3 \times 0 = 0$

DECREASING A FACTOR BY 1 DECREASES THE PRODUCT BY 3.

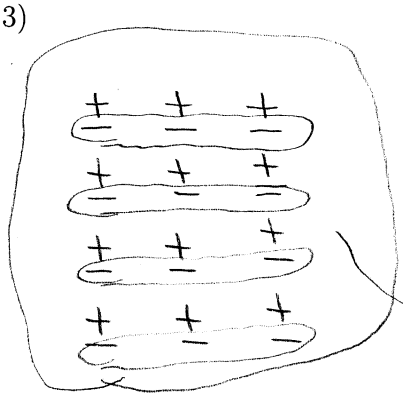
CONTINUING...

$3 \times (-1) = -3$
$3 \times (-2) = -6$
$3 \times (-3) = -9$
$3 \times (-4) = -12$
$3 \times (-5) = -15$

(b) $-4 \times (-3)$

CHARGES

START WITH ZERO



TAKE OUT 4 GROUPS OF 3-'s

TAKE OUT

LEFT WITH 12 +'s

$-4 \times (-3) = 12$

10. (1 point) What is the additive inverse of $x - y + 5$ and why?

$x - y + 5 = x + (-y) + 5$

ADDITIVE INVERSE IS $\boxed{-x + y + (-5)}$

OR $-x + y - 5$, BECAUSE THIS ADDED TO THE ORIGINAL EXPRESSION GIVES ZERO.

11. (5 points) Test the number 28,586,580 for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, and 11. Explain your reasoning!

2: Yes, because num ends with 0

3: $2+8+5+8+6+5+8 = 42$ and $3 \mid 42 \Rightarrow$ Yes

4: $4 \mid 80 \Rightarrow$ Yes

5: Yes, because num ends with 0

6: Yes, because num is divisible by both 2 and 3

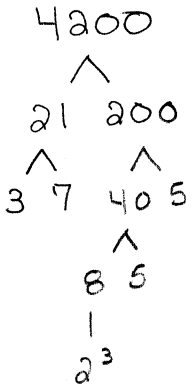
8: No, because $8 \nmid 580$

9: No, because sum of digits is 42 and $9 \nmid 42$

10: Yes, because num ends with 0

11: $0+5+8+8 = 21$
$8+6+5+2 = 21$
$21-21 = 0$ and 0 is divisible by 11
$\Rightarrow$ Yes

12. (3 points) Find the prime factorization of 4200. Then determine its number of positive divisors.



$4200 = 2^3 \cdot 3 \cdot 5^2 \cdot 7$

# of divisors is

$4 \times 2 \times 3 \times 2 = 48$

13. (2 points) Describe a test that could be used to determine if an integer is divisible by 20. Explain how you know your test is valid.

An integer is divisible by 20 if and only if it is divisible by both 4 and 5.

Or, an integer is divisible by 20 if and only if the integer formed by last 2 digits is divisible by 20.