Math 200 - 1st Final Exam

May 2, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary. Multiple choice problems are worth 0, 1, or 2 points depending on your answer and the work shown.

1. A student said that all odd numbers are prime. Which one of the following is a counterexample?

   (a) 2

   (b) 7

   (c) 9 ← AN ODD NUMBER THAT IS NOT PRIME

   (d) 12

2. Compute the sum: $1+2+3+4+\cdots+1723+1724+1725$

   (a) 1,488,675

   (b) 2,977,350

   (c) 5,182

   (d) 1,487,812.5

   $1725+1724+\cdots+3+2+1$

   1725 groups of 1726 / 2 = 2977350 / 2

   = 1,488,675

3. Which one of the following sets is NOT well defined?

   (a) The set of all natural numbers

   (b) The set of all integers less than -12

   (c) The set of all whole numbers less than 0

   (d) The set of all big integers WHAT IS A BIG NUMBER ?

4. The first term of a **geometric sequence** is 4 and its ratio is 7. Which one of the following is its third term?

   (a) 1372

   (b) 196

   (c) 18

   (d) 25

   $4, 4\times7, 4\times7^2, \ldots$

   $4, 28, 196, \ldots$

5. If $A=\{2,4,6\}$ and $B=\{1,3\}$, then which one of the following is $n(A\times B)$?

   (a) 6

   (b) $(1,6)$

   (c) $\{1,2,3,4,6\}$

   (d) 5

   $N(A\times B) = N(A)\times N(B)$

   $= 3\times 2$

6. Choose the multiplication model that best fits the following problem situation: *A restaurant offers combo meals with choices of 10 different entrees and 6 different appetizers. How many different combo meals are offered?*

   (a) array
   (b) missing factor
   (c) repeated addition
   (d) Cartesian product

7. Which one of the following is the LCM of 6 and 10?

   (a) 6
   (b) 2
   (c) 60
   (d) 30

   10, 20, 30, ← Also a multiple of 6

8. Which one of these is NOT a correctly written base-seven numeral?

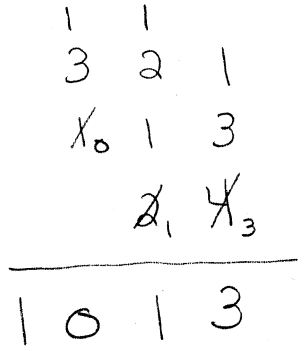
   (a) 66006_{seven}
   (b) 1110011_{seven}
   (c) 5_{seven}
   (d) 17_{seven} ← No digit 7 in base-7

9. Let Z be the set of integers. Which one of the following is NOT an element of the set $A = \{x \mid x = 3n + 1 \text{ where } n \in Z\}$.

   (a) 10 ← $3(3)+1$
   (b) -6
   (c) -2 ← $3(-1)+1$
   (d) 7 ← $3(2)+1$

10. Compute the sum: $321_{\text{five}} + 113_{\text{five}} + 24_{\text{five}}$

    (a) 1013_{five}
    (b) 458_{five}
    (c) 513_{five}
    (d) 423_{five}



11. Choose the division model that best fits the following problem situation: *Ms. Smith has 24 markers that she must divide into 6 groups of equal size. How many markers will there be in each group?*

(a) missing factor

(b) repeated subtraction

(c) Cartesian product

(d) set partition

12. If x is a negative number, then what can be said about $-2x$?

(a) $-2x$ is negative

(b) $-2x$ is positive

(c) $-2x$ could be zero

(d) More information is needed

13. Suppose $A = \{x, y, z\}$ and $A \sim B$. Which one of the following must be true?

(a) $x \in B$

(b) $n(B) = 3$

(c) $B = \overline{A}$

(d) $A \cup B = A$

14. Which one of the following facts demonstrates the associative property of multiplication?

(a) $2(x + 1) + 4 \cdot 5 = 2(x + 1) + 5 \cdot 4$

(b) $(6x + 10) = 2(3x + 5)$

(c) $2 \cdot (6 \cdot 7) + 1 = (2 \cdot 6) \cdot 7 + 1$

(d) $(4 + 1) + 2 = 4 + (1 + 2)$

15. Which one of the following divisibility tests is incorrect?

(a) A whole number is divisible by 3 if the sum of its digits is divisible by 3.

(b) A whole number is divisible by 6 if is divisible by both 2 and 3.

(c) A whole number is divisible by 8 if the number formed by its last three digits is divisible by 8.

(d) A whole number is divisible by 9 if the number formed by its last 2 digits is divisible by 9.

16. (5 points) Clearly state the steps of the problem-solving process (in order). Then choose any one step and state two different strategies associated with that step.

See page 4 of textbook.

① Understand the Problem

② Devise a Plan

③ Carry out the Plan

④ Look Back

i) Check your result in the original wording of the problem

ii) Generalize

17. (5 points) Use a model to illustrate and compute each of the following.

(a) -3×2

Pattern

$3 \times 2 = 6$

$2 \times 2 = 4$

$1 \times 2 = 2$

$0 \times 2 = 0$

Factor decreases by 1, Product decreases by 2

Continuing...

$-1 \times 2 = -2$

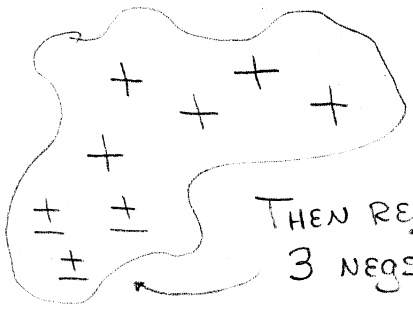
$-2 \times 2 = -4$

$-2 \times 3 = -6$

(b) $5 - (-3)$

Charges

Start with +5



Then remove 3 negs.

Left with 8 +'s

$5 - (-3) = 8$.

(c) $-4 - 7$

Number Line

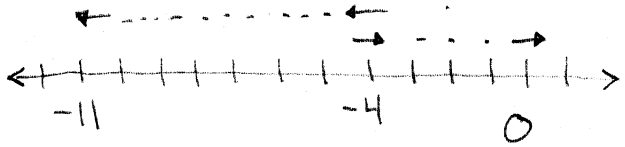
① Start at zero facing right

② Back up 4

③ Turn around

④ Go forward 7

⑤ End at 4 − 11

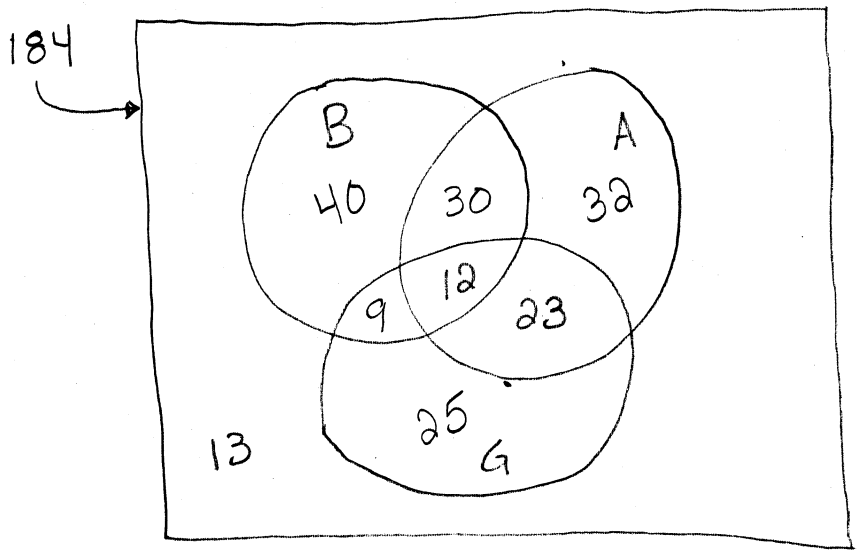


$-4 - 7 = -11$

18. (5 points) 184 children were asked to name the fruits they often eat. The following results were obtained:

- 91 said bananas
- 97 said apples
- 69 said grapes
- 42 said bananas and apples
- 21 said bananas and grapes
- 35 said apples and grapes
- 12 said bananas, apples, and grapes

Organize this data in a three-set Venn diagram. How many children surveyed named none of these three fruits?



$40 + 30 + 32 + 9 + 12 + 23 + 25$

$= 171$

$184 - 171 = 13$

13 CHILDREN NAMED NONE OF THE THREE FRUITS

19. (5 points) Use any method to find the GCD of 630 and 6615.

EUCLIDEAN ALGORITHM ...

$$\begin{array}{r} 10 \\ 630\overline{)6615} \\ 6300 \\ \hline 315 \end{array}$$

$$\begin{array}{r} 2 \\ 315\overline{)630} \\ 630 \\ \hline 0 \end{array}$$

$GCD(6615, 630) = GCD(630, 315)$

$= GCD(315, 0) = \boxed{315}$

20. (5 points) Carefully explain how you would efficiently determine whether 839 is prime? Is it?

$\sqrt{839} \approx 28.965$

THE GREATEST PRIME LESS THAN OR EQUAL TO $\sqrt{839}$ IS 23.

TO TEST WHETHER 839 IS PRIME, WE CHECK IT FOR DIVISIBILITY BY ALL PRIMES UP THROUGH 23:

2, 3, 5, 7, 11, 13, 17, 19, 23

839 IS NOT DIVISIBLE BY ANY OF THESE. IT HAS TO BE PRIME.

21. (5 points) Write the numeral 353_{six} in expanded form. Then list the next five base-six numerals.

$353_{SIX} = 3 \times 6^2 + 5 \times 6^1 + 3 \times 6^0$

NEXT 5 ARE...

354_{SIX}

355_{SIX}

400_{SIX}

401_{SIX}

402_{SIX}

22. (5 points) Clearly state the rule for adding two integers with opposite signs. Give an example that illustrates your rule.

To add two numbers with opposite signs, subtract their absolute values, least from greatest. Then give that result the sign of the original addend with the greatest absolute value.

Ex $7 + (-9) = -(9-7) = -2$

23. (5 points) Suppose U is the set of all Americans, A is the set of all American smokers, and B is the set of all Americans with health problems. **Describe a person** who is an element of each of the following sets.

(a) $A \cap B$

An element of $A \cap B$ is an American smoker with health problems

(b) $A \cap \overline{B}$

... an American smoker who does not have health problems

(c) $B - A$

... an American with health problems who is not a smoker

24. (5 points) Write a whole number with five different digits. Then use divisibility tests to test for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, and 11.

12345

2: No, because last digit 5 is not divisible by 2

3: Yes, because $1+2+3+4+5=15$ and 15 is divisible by 3

4: No, because 45 is not divisible by 4

5: Yes, because ends in 5

6: No, divisible by 3 but not 2

8: No, because 345 is not divisible by 8

(In fact, original number is not even divisible by 2.)

9: No, because $1+2+3+4+5=15$ and 15 is not divisible by 9

10: No, doesn't end in zero

11: No, $(5+3+1)-(4+2)=9-6=3$

and 3 is not divisible by 11.