Math 206 - Test 1

February 17, 2010

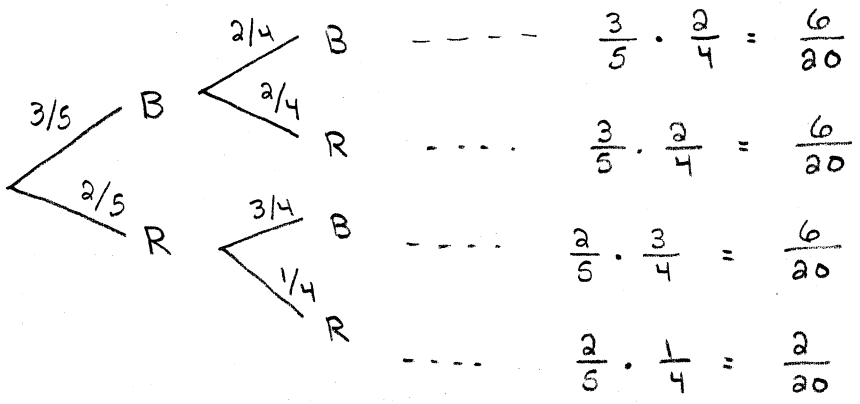
Name key

Score _____

Show all work. Supply explanations where necessary.

1. (8 points) A box contains three blue marbles and two red marbles. A marble is selected at random (without replacement), and its color is recorded. Another marble is then selected, and its color is also recorded. The sample space for this experiment is written $S = \{BB, BR, RB, RR\}$.

   (a) Sketch the probability tree diagram that models the experiment.



   (b) Find the probability of each outcome in S.

$P(BB) = \frac{6}{20}$ $\quad$ $P(RB) = \frac{6}{20}$

$P(BR) = \frac{6}{20}$ $\quad$ $P(RR) = \frac{2}{20}$

   (c) What is the probability of selecting a red marble on the 2nd draw?

$$P(BR) + P(RR) = \frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5}$$

   (d) Are the probabilities you've given above theoretical probabilities or experimental probabilities? What is the difference between the two types of probabilities?

THEORETICAL PROBABILITIES.

EXPERIMENTAL PROBS ARE ASSIGNED BY PERFORMING THE EXPERIMENT AND OBSERVING OUTCOMES.

THEO PROBS ARE ASSIGNED UNDER IDEAL CONDITIONS.

2. (3 points) Marty can build a patio deck by himself in 15 hours. His wife can build the same patio deck by herself in 25 hours. How long will it take them to build the deck if they work together.

$$\frac{1 \text{ DECK}}{15 \text{ HRS}} + \frac{1 \text{ DECK}}{25 \text{ HOURS}} = \frac{5 \text{ DECKS}}{75 \text{ HRS}} + \frac{3 \text{ DECKS}}{75 \text{ HOURS}}$$

$$= \frac{8 \text{ DECKS}}{75 \text{ HOURS}} = \frac{1 \text{ DECK}}{X} \Rightarrow X = \frac{75}{8}$$

$$= \boxed{9.375 \text{ HRS}}$$

3. (2 points) Solve each problem.

   (a) If 5 cans of peaches cost \$2.95, how much do 8 cans cost?

$$\frac{\$2.95}{5} = \$0.59 \text{ EACH}$$

$$8 \times \$0.59 = \boxed{\$4.72}$$

   (b) After working for 9 months, Sarah earned 6 vacation days. At this rate, how many does does she earn per year?

$$\frac{6 \text{ DAYS}}{9 \text{ MONTHS}} = \frac{X}{12 \text{ MONTHS}}$$

$$9X = 72 \Rightarrow X = \boxed{8 \text{ DAYS}}$$

4. (1 point) When Joe walked into the restaurant, he noticed that there were 4 times as many women as men. Let W and M represent the numbers of women and men, respectively. Write an equation that relates W and M.

$$\frac{W}{M} = \frac{4}{1} \quad \text{OR} \quad 4M = W$$

5. (3 points) Consider the number $\frac{1}{103}$.

(a) Circle each set of numbers to which this number belongs.

Whole Numbers Integers (Rational Numbers)

Irrational Numbers (Real Numbers)

(b) Does the decimal form of this number repeat, terminate, or neither? How do you know?

$\frac{1}{103}$ HAS A REPEATING DECIMAL FORM.

$\frac{1}{103}$ IS IN LOWEST TERMS, AND SINCE 103 IS PRIME, THE DENOM CANNOT BE WRITTEN AS A PRODUCT OF 2's & 5's.

(c) To twenty-five decimal places, $\frac{1}{103} = 0.0097087378640776699029126$. Are you surprised that it has not terminated or begun to repeat after this many places? Explain your answer.

I AM NOT SURPRISED AT ALL.

THE DECIMAL FORM WILL DEFINITELY REPEAT, BUT THE REPETEND MIGHT HAVE UP TO 102 DIGITS.

6. (3 points) Show how the decimal form of $\frac{3}{2^3 \cdot 5}$ can be obtained without using division.

$$\frac{3}{2^3 \cdot 5} \cdot \frac{5^2}{5^2} = \frac{3 \cdot 25}{2^3 \cdot 5^3} = \frac{75}{1000} = \boxed{0.075}$$

7. (3 points) If you know a family has two children, and a boy answers when you knock on the door, what is the probability that the other child is a boy? Carefully explain your reasoning.

PROBABILITY IS $\frac{1}{2}$. IF WE ASSUME EACH GENDER IS EQUALLY LIKELY AND THE BIRTHS ARE INDEPENDENT, THEN PROB OF BOY IS ALWAYS $\frac{1}{2}$.

8. (3 points) Box #1 contains the numbers 1, 3, and 5. Box #2 contains the numbers 2, 4, 6, and 8. Box #3 contains the numbers 1 and 2. A box is selected at random, and a number is selected from that box. The ratio of the box number to the selected number is then formed. What is the probability that your ratio has a terminating decimal form?

ALL RATIOS ARE TERMINATING OR REPEATING.

THERE ARE ONLY TWO WAYS TO GET A REPEATING DECIMAL:

(#1, 3) AND (#2, 6)

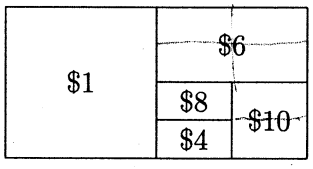
PROB IS $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

PROB IS $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

$\Rightarrow$ PROB OF REPEATING DECIMAL IS $\frac{1}{9} + \frac{1}{12} = \frac{7}{36}$

PROB OF TERMINATING DECIMAL IS $1 - \frac{7}{36} = \boxed{\frac{29}{36}}$

9. (3 points) A dart lands at random on the board shown below. The thrower wins the amount of money associated with the dart's location. What is the probability that the thrower wins more than \$7? Briefly explain your reasoning.



IF WE MAKE EQUAL-SIZED RECTANGLES, THERE ARE 16 OF THEM.

3 WOULD BE ASSOCIATED WITH MORE THAN \$7.

PROB IS $\frac{3}{16}$.

10. (3 points) A box contains 25% black balls and 75% white balls. The same number of black balls as was in the box are added (so the new number of black balls is twice the original number). A ball is now draw from the box at random. What is the probability that it is black?

25 BLACK
75 WHITE

→

50 BLACK
75 WHITE

PROB OF DRAWING BLACK IS NOW $\frac{50}{125} = 40\%$

11. (3 points) Obtain a die from your instructor. Do not count the number of colored faces until it is necessary. What color is on your die? ANSWERS VARY

(a) Determine the experimental probability of your die landing colored side up. Explain your reasoning.

ROLL DIE MANY TIMES. EXP PROB = $\frac{\#\text{ OF TIMES LANDED COLORED SIDE UP}}{\#\text{ OF TIMES ROLLED}}$

(b) Determine the theoretical probability of your die landing colored side up. Explain your reasoning.

THEO PROB = $\frac{\#\text{ OF COLORED SIDES}}{6}$

12. (3 points) Suppose A, B, and C are events such that $P(A) = 0.3$, $P(B) = 0.6$, and $P(C) = 0$. Find each probability.

(a) $P(\overline{C})$ = $1 - P(C) = 1 - 0 =$ 1

(b) $P(A \cup B)$ if A and B are mutually exclusive

$= P(A) + P(B) - P(A \cap B) = 0.3 + 0.6 - 0 =$ 0.9

(c) $P(A \cap B)$ if $P(A \cup B) = 0.5$

$0.5 = 0.3 + 0.6 - P(A \cap B)$

$\Rightarrow P(A \cap B) =$ 0.4

13. (3 points) Biologists studying Australian bandicoots captured, tagged, and released 275 bandicoots. In the same area, a later survey team captured 200 bandicoots, 45 of which were tagged. Estimate the bandicoot population in the survey area.

$$\frac{\text{Tagged}}{\text{Total}} = \frac{275}{x} = \frac{45}{200}$$

$$45x = 55000$$

$$x = 1222.\overline{2}$$

Pop is about 1222

14. (4 points) A letter is selected at random from the word *MISSISSIPPI*.

(a) Give examples of two events, A and B, that are equally likely.

A = event of drawing I

B = event of drawing S

(b) Give examples of two events, C and D, that are mutually exclusive.

C = event of drawing M or S

D = event of drawing I or P

(c) Give examples of two events, X and Y, that are NOT mutually exclusive.

X = event of drawing S

Y = event of drawing a letter from 2nd half of alphabet

(d) Give an example of an event with probability 1.

Event of drawing M, I, S, or P.

15. (5 points) Do either ONE of the following problems.

(a) Design a simulation that could be used to estimate the solution of the following problem.

> Police estimate the probability of a driver's wearing the shoulder belt is about 0.70. If the police randomly stop 10 motorists, what is the probability that they will find 6 or more who aren't wearing belts?

Perform five trials of your simulation and explain how you would use your results to answer the question.

USE A RANDOM DIGIT TABLE. A TRIAL WILL CONSIST OF SELECTING A BLOCK OF 10 DIGITS. 1,2,3,4,5,6,7 REPRESENT PEOPLE WEARING BELTS, WHILE 8,90 REPRESENT THOSE NOT WEARING BELTS. A SUCCESS IS A TRIAL WHERE 6 OR MORE 8,9,0 COME UP.

TRIAL #1 : 0220210807 FAILURE

TRIAL #2 : 8488373916 FAILURE

TRIAL #3 : 0282449034 FAILURE

TRIAL #4 : 4264716593 FAILURE

TRIAL #5 : 4131977275 FAILURE

THERE WERE NO SUCCESSES IN MY TRIALS. 0/5

THIS PROB IS PROBABLY PRETTY SMALL.

(b) Design a simulation that could be used to estimate the solution of the following problem.

> What is the probability that in a group of five people chosen at random, at least two will have birthdays in the same month?

Perform five trials of your simulation and explain how you would use your results to answer the question.

USE A 12-SIDED DIE. ROLL IT 5 TIMES. IF THE SAME NUMBER COMES UP TWICE, THEN YOU'VE FOUND TWO PEOPLE WITH THE SAME BIRTH MONTH.

TRIAL #1 : 1, 7, 3, 10, 8 FAILURE

TRIAL #2 : 1, 3, 4, 1, 1 SUCCESS

TRIAL #3 : 3, 1, 9, 1, 2 SUCCESS

TRIAL #4 : 9, 11, 12, 2, 7 FAILURE

TRIAL #5 : 11, 2, 7, 12, 12 SUCCESS

$$\frac{\#\text{ OF SUCCESSES}}{\#\text{ OF TRIALS}} \approx \text{PROB}$$

$$\text{PROB} \approx \frac{3}{5}$$