Math 206 - 1st Final Exam

May 5, 2010

Name key

Score _____

Show all work. Supply explanations where necessary. Use only a compass and a straightedge for constructions. For each construction, the steps you follow must be apparent.

1. (5 points) An experiment consists of tossing a penny, a nickel, and a dime simultaneously.

   (a) List the elements of an equally likely sample space. (There are 8 elements in the sample space.) I'LL USE H & T FOR HEAD & TAIL. I'LL WRITE MY ELEMENTS IN PENNY, NICKEL, DIME ORDER.

   $S = \{HHH, THH, HTH, TTH, HHT, THT, HTT, TTT\}$

   (b) List the event of obtaining a tail on the nickel.

   $\{HTH, TTH, HTT, TTT\}$

   (c) What is the probability of obtaining a tail on the nickel?

   $\frac{4}{8}$ OR $\frac{1}{2}$

   (d) Is your answer for part (c) an experimental probability or a theoretical probability?

   THEORETICAL

2. (4 points) The mean score for 25 of 27 tests is 80. The other two scores are 30 and 35.

   (a) What is the mean of all twenty-seven test scores?

   $$\frac{25(80) + 30 + 35}{27} = 76.48$$

   (b) Is it possible to find the median test score with the given information? Explain.

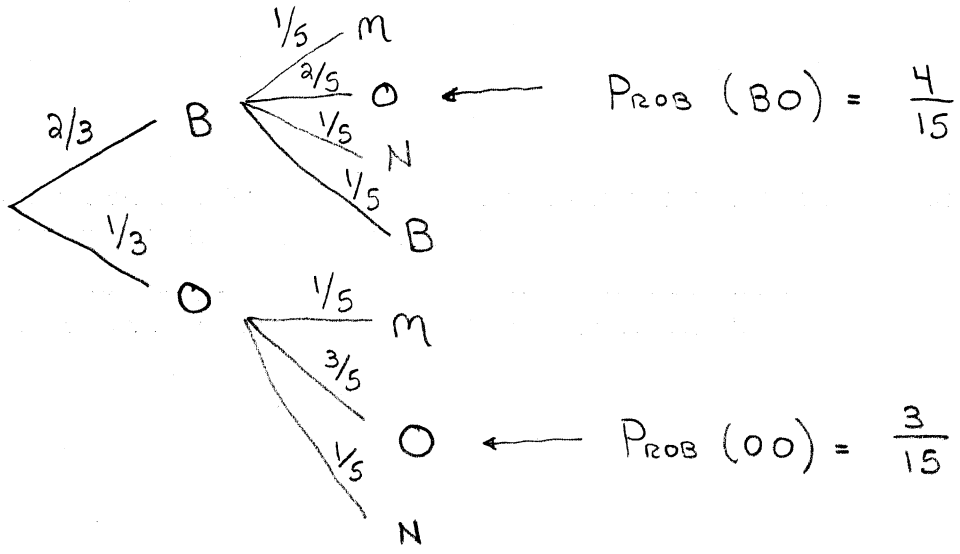
   NO, ONCE THE SCORES ARE ARRANGED NUMERICALLY, THE MEDIAN WOULD BE THE 14TH SCORE. WE'D NEED MORE INFO TO GET THE MEDIAN.

3. (4 points) A letter is selected at random from the first box and placed into the second box. A letter is then selected from the second box.

B O B

M O O N

What are the odds in favor of selecting the letter "O" from the second box?



$\text{PROB}(BO) = \frac{4}{15}$

$\text{PROB}(OO) = \frac{3}{15}$

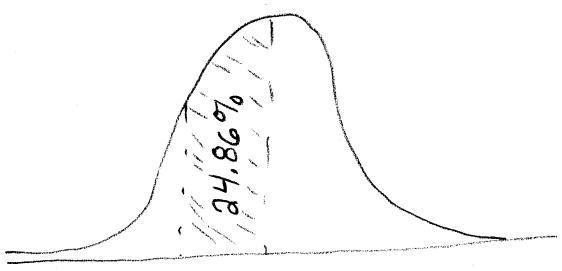
$\text{PROB OF O ON 2ND} = \frac{7}{15}$

ODDS ARE 7/8

4. (4 points) A certain brand of light bulb has a mean lifetime of 1700 hr with a standard deviation of 150 hr. Assuming the lifetimes are normally distributed, about how many in an order of 5000 will have a lifetime of between 1600 hr and 1700 hr?

$Z_{1700} = 0$ (THAT'S THE MEAN!)

$Z_{1600} = \frac{1600-1700}{150} = -0.67$ Look up 0.67 to get 24.86%



24.86% OF 5000

= 1243

5. (4 points) A game consists of rolling a regular die with prizes awarded as follows:

- Roll a 1 and win \$5 Prob $\frac{1}{6}$
- Roll a 2, 4, or 6 and win \$2 Prob $\frac{3}{6}$
- Roll a 3 or 5 and win \$1 Prob $\frac{2}{6}$

If the game costs \$3 to play, how much money should one expect to gain or lose on average with each play?

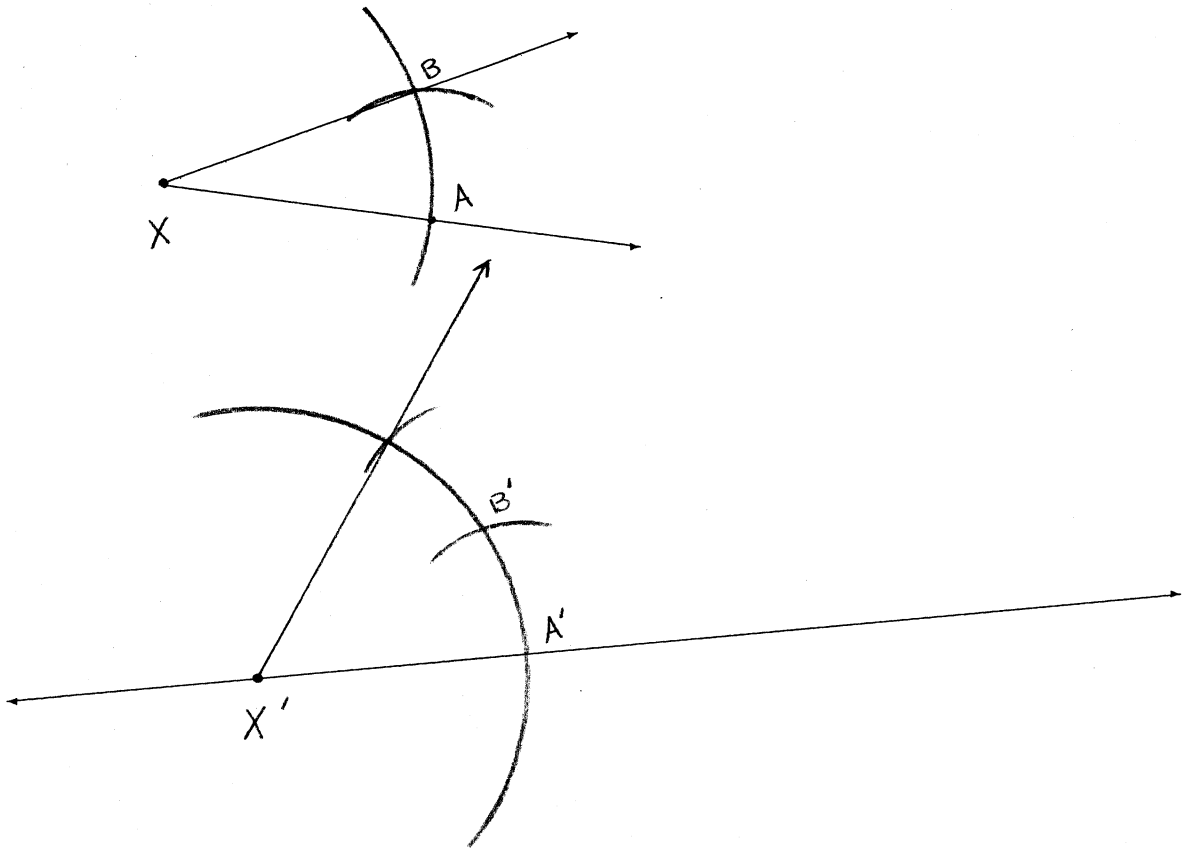
Expected value $= \$5\left(\frac{1}{6}\right) + \$2\left(\frac{3}{6}\right) + \$1\left(\frac{2}{6}\right)$

$= \$\left(\frac{5}{6} + \frac{6}{6} + \frac{2}{6}\right) = \$\left(\frac{13}{6}\right) = \$2.1\overline{6}$

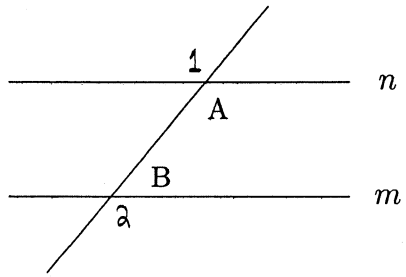
$\approx \$2.17$

Expect to lose $\$0.8\overline{3}$ per play.

6. (5 points) Use a compass and a straightedge to construct an angle whose measure is twice that of the angle shown below. Place the initial side of your new angle on the line below.



7. (4 points) In the following figure $n \| m$.



(a) In the figure, label a pair of alternate exterior angles.

$\angle 1$ AND $\angle 2$ ARE ALT EXTERIOR.

(b) Find x if $m(\angle A) = 2x + 4$ and $m(\angle B) = 3x - 14$.

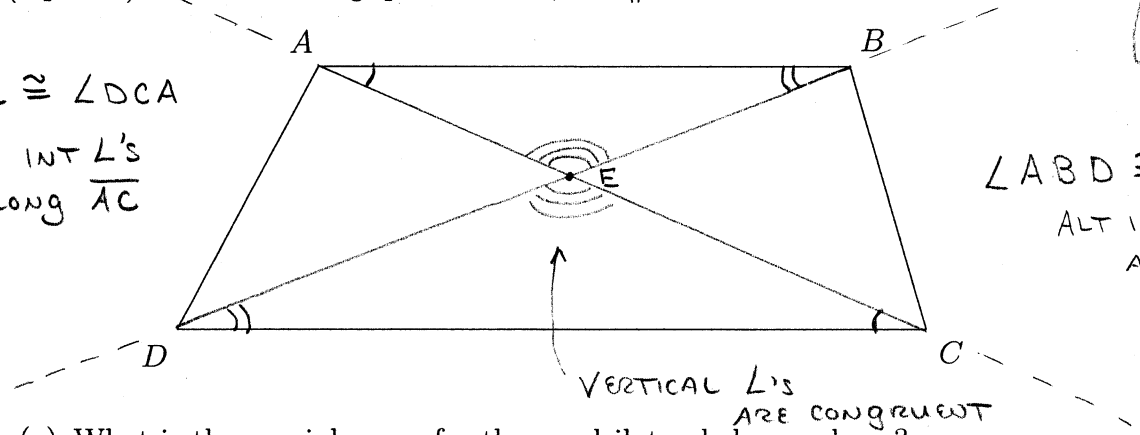
$\angle A$ & $\angle B$ ARE SUPPLEMENTARY.

$2x + 4 + 3x - 14 = 180$ $\quad 5x - 10 = 180^\circ$

$5x = 190^\circ$

$x = 38^\circ$

8. (4 points) In the following quadrilateral, $\overline{AB} \| \overline{CD}$.



$\angle BAC \cong \angle DCA$ ALT INT $\angle$'S ALONG $\overline{AC}$

$\angle ABD \cong \angle CDB$ ALT INT $\angle$'S ALONG $\overline{BD}$

VERTICAL $\angle$'S ARE CONGRUENT

(a) What is the special name for the quadrilateral shown above?

TRAPEZOID

(b) Sketch the diagonals of the quadrilateral. Once you have sketched the diagonals, find two similar triangles within the figure. How do you know they are similar?

$\triangle AEB \sim \triangle CED$ BY AAA

SEE ABOVE.

9. (4 points) An interior angle of a convex hexagon measures 100°. All of the other interior angles have equal measures (but not 100°). Find the measures of the interior and exterior angles.

INTERIOR ANGLES ADD UP TO $(6-2)\,180° = 4(180°) = 720°$

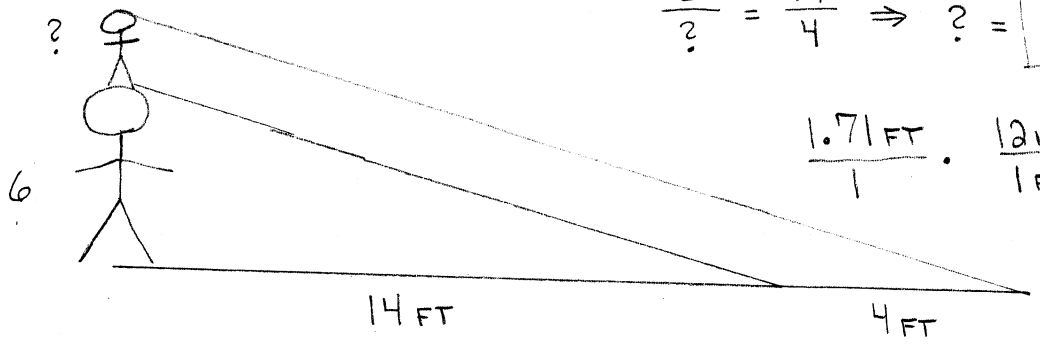
ONE MEASURES $100° \Rightarrow$ OTHERS ADD UP TO $620°$

EACH MEASURES $\frac{620°}{5} = 124°$

INTERIOR ANGLES: $100°$, 5 MEASURE $124°$

EXTERIOR ANGLES: $80°$, 5 MEASURE $56°$ ← TOTAL OF $360°$

10. (5 points) On Tuesday at 4pm, zookeeper Joe, who is exactly 6 ft tall cast 14-ft shadow. A second later, a monkey jumped onto his head and the shadow grew to 18 ft. How tall is the monkey? Round your answer to the nearest hundredth of a foot, and then convert that answer to centimeters.

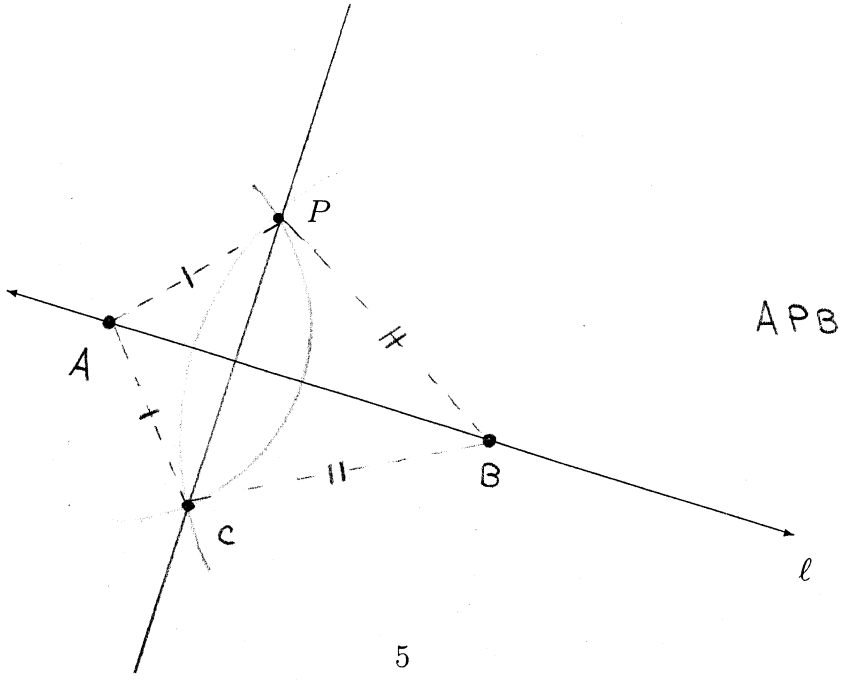


$\frac{6}{?} = \frac{14}{4} \Rightarrow ? = 1.71$ FT

$\frac{1.71\text{ FT}}{1} \cdot \frac{12\text{ IN}}{1\text{ FT}} \cdot \frac{2.54\text{ cm}}{1\text{ IN}} =$

52.12 cm

11. (4 points) Use only a compass and straightedge to construct a line through P perpendicular to ℓ.



APBC IS A KITE

12. (6 points) Suppose A, B, and C are events such that $P(A) = 0.57$, $P(B) = 0.24$, and $P(C) = 0.99$.

(a) Find $P(\overline{C})$.

$P(C) + P(\overline{C}) = 1 \Rightarrow P(\overline{C}) = \boxed{0.01}$

(b) Find $P(A \cup B)$ if A and B are mutually exclusive.

$P(A \cap B) = 0$

$\Rightarrow P(A \cup B) = P(A) + P(B) = 0.57 + 0.24 = \boxed{0.81}$

(c) Find $P(A \cap B)$ if $P(A \cup B) = 0.5$.

$0.5 = 0.57 + 0.24 - P(A \cap B)$

$0.5 = 0.81 - P(A \cap B) \Rightarrow P(A \cap B) = \boxed{0.31}$

(d) Is it possible that C and B are mutually exclusive? Explain.

No, because $P(C) + P(B) = 1.23 > 1$

13. (6 points) Fill in the blank with the correct word.

(a) Lines that share a common point are called CONCURRENT lines.

(b) A curve made up entirely of straight line segments is called a POLYGONAL curve.

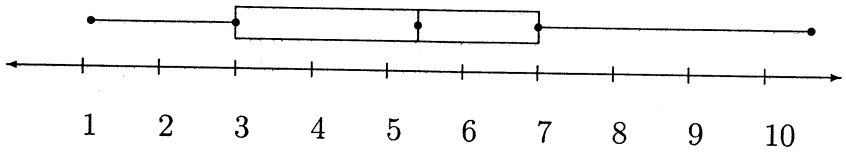
(c) Points that lie on the same line are said to be COLLINEAR.

(d) A SIMPLE curve is a curve that does not cross itself.

(e) The angle between two planes is called a DIHEDRAL angle.

(f) Two coplanar lines that have exactly one point in common are called INTERSECTING lines.

14. (6 points) The boxplot shown below describes a certain collection of data. Find approximate values for the median, lower and upper quartiles, and the interquartile range. Based on your approximations, what would be the cutoff values for outliers?



MEDIAN $\approx$ 5.4

$Q_1 \approx$ 3

$Q_3 \approx$ 7

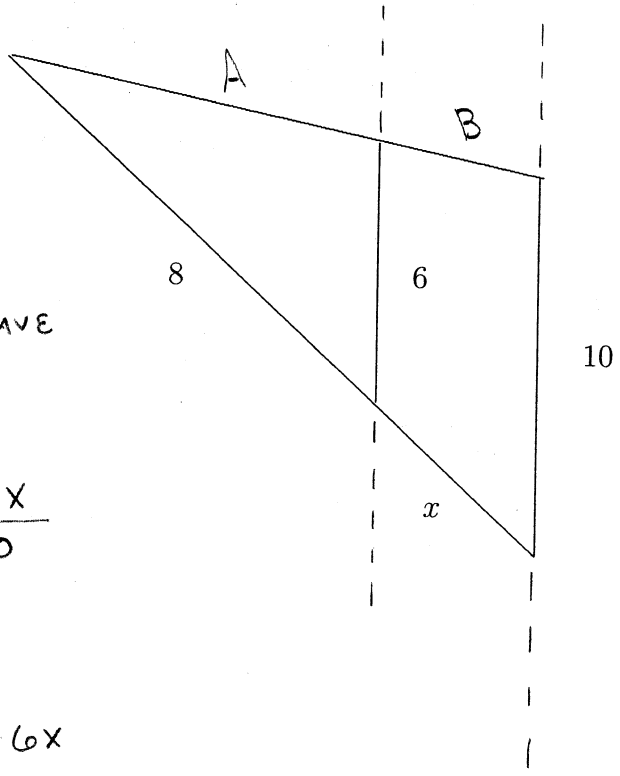
IQR $\approx 7-3 =$ 4

$1.5 \times IQR = 6$

$Q_1 - 1.5 \times IQR = 3-6 = -3$

$Q_3 + 1.5 \times IQR = 7+6 = 13$

15. (4 points) Trudy was trying to find the value of x in the following diagram. She knew that there were similar triangles shown in the diagram, so she set up and solved the following proportion: $\frac{6}{10} = \frac{8}{x}$. Her answer didn't seem quite right, but she knew that these diagrams are not always drawn to scale. Is Trudy correct? Carefully explain your reasoning.



TRUDY SHOULD HAVE

$\frac{8}{6} = \frac{8+x}{10}$

OR

$80 = 48 + 6x$

$32 = 6x$

$x = 5.\overline{3}$

THE SIDES CUT BY THE PARALLEL LINES FROM PROPORTIONAL SEGMENTS:

$\frac{8}{A} = \frac{x}{B}$

THIS RESULT DOESN'T APPLY TO THE SIDES WITH LENGTHS 6 & 10.

16. (6 points) For each of the following situations, tell which type of graph would best display the data. Choose from *line plot*, *bar graph*, *histogram*, *line graph*, *stem-and-leaf plot*, or *circle graph*. Give a brief explanation to support your answer.

(a) Alberto has been keeping track of gas prices. He would like to make a graph showing how the prices have changed over the last two weeks.

TRENDS IN TIME ARE BEST SHOWN WITH **LINE GRAPHS.**

(b) A candidate's campaign fund has come from donations in six different categories. The candidate would like to make a graph showing how the entire fund is divided up among the six categories.

CIRCLE GRAPHS ARE USED FOR SHOWING PORTIONS OF A WHOLE.

(c) The National Center for Health Statistics keeps detailed records on the births and deaths of US residents. What type of graph would best display the ages at death of US residents in 2007?

AGES ARE CONTINUOUS DATA. IF THE AGES ARE GROUPED INTO INTERVALS, A **HISTOGRAM** IS BEST.