Math 206 - Test 1

February 15, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) In class we studied a technique that could be used without doing division to determine whether a reduced fraction has a repeating or a terminating decimal form.

   (a) Describe the technique. FIND THE PRIME FACTORIZATION OF THE DENOMINATOR. THE DECIMAL FORM IS TERMINATING IF THE ONLY PRIME FACTORS ARE 2's AND/OR 5's.

   (b) Use the technique to determine which (if any) of these fractions has a repeating decimal form.

   BOTH ARE IN LOWEST TERMS →

   $\frac{98}{625}$: $625 = 25 \cdot 25 = 5 \cdot 5 \cdot 5 \cdot 5 = 5^4$

   $\frac{17}{56}$: $56 = 8 \cdot 7 = 2 \cdot 4 \cdot 7 = 2 \cdot 2 \cdot 2 \cdot 7 = 2^3 \cdot 7$

   BY OUR TECHNIQUE, 17/56 REPEATS.

   (c) Give an example to show that the technique can fail if the fraction is not first reduced to lowest terms.

   $\frac{6}{12} = 0.5$ IT TERMINATES, BUT THE PRIME FACTORIZATION OF 12 IS $3 \cdot 2 \cdot 2$.

2. (3 points) Believe it or not, *zenzizenizenic* is an actual word. Suppose a letter is selected at random from this word.

   (a) Give a possible sample space for this experiment.

   $S = \{z, e, n, i, c\}$

   (b) Is your sample space uniform? Explain.

   NO, THE ELEMENTS OF S ARE NOT EQUALLY LIKELY BECAUSE THEY OCCUR DIFFERENT NUMBERS OF TIMES IN THE WORD.

   (c) State an event that has probability less than 0.5, and give the probability of your event.

   $\{c\}$ HAS PROBABILITY $\frac{1}{14}$ WHICH IS MUCH LESS THAN 0.5

3. (3 points) A jar is filled with colored marbles. The probability of selecting a red marble is 21/80. Is it possible that the probability of selecting a blue marble is 3/4? Explain.

No, IF SO, THE PROBABILITY OF SELECTING A RED OR BLUE MARBLE WOULD BE $\frac{21}{80} + \frac{3}{4} = \frac{81}{80}$.

PROBABILITIES CANNOT EXCEED 1.

4. (3 points) Solve each of the following. Use the unit rate approach for one, the scale factor approach for one, and standard cross multiplication for one. Be sure to state which approach is which.

   (a) If there are 3 cars for every 8 students at the high school, how many cars are there for 1600 students?

SCALE FACTOR... 200 TIMES AS MANY STUDENTS

$\Rightarrow$ 200 TIMES AS MANY CARS

$3 \times 200 =$ 600 CARS

   (b) On a certain map, 5 inches corresponds to 20 miles. How many miles correspond to 13 inches?

UNIT RATE ... 4 MILES PER INCH

$\Rightarrow$ 13 INCHES CORRESPOND TO 52 MILES

   (c) If Marty paid \$19 for 7 DVD's, how much would 13 DVD's cost him?

CROSS MULTIPLY... $\frac{19}{7} = \frac{?}{13} \Rightarrow ? = \frac{19 \cdot 13}{7} \approx$ \$35.29

5. (2 points) In order to solve her math problem, Sophia set up the following proportion: $\frac{45}{72} = \frac{x}{100}$. State the other three equivalent proportions that Sophia could just as easily use.

⓪ $\frac{45}{72} = \frac{X}{100}$

② $\frac{45}{X} = \frac{72}{100}$

① $\frac{72}{45} = \frac{100}{X}$

③ $\frac{X}{45} = \frac{100}{72}$

6. (3 points) Give examples of two irrational numbers and one rational number that lie between 3/5 and 5/8.

$\frac{3}{5} = 0.6$

$\frac{5}{8} = 0.625$

TWO IRRATIONALS :

$0.61661666166661\ldots$

$0.62020020000\ldots$

ONE RATIONAL :

0.615

7. (3 points) Each situation below describes a multistage experiment. Determine the best number of stages for each.

   (a) A small bag contains a variety of golf tees. Three tees are selected at random.

   3 STAGES

   (b) Letters of the word *ovoviviparous* are selected at random without replacement in an attempt to spell the word VAPOR.

   5 STAGES

   (c) A marble is selected from one jar and placed into a second jar. Then a marble is selected from the second jar.

   2 STAGES

8. (2 points) Suppose the wicked witch of the east is hanging out at a random location in her big yard which measures 120 ft by 310 ft. Dorothy Gale's little house measures 12 ft by 10 ft, and a tornado is about to hurl the house into the witch's yard. What is the probability that the witch is smashed by the falling house?

$$\text{PROB IS } \frac{\text{AREA OF HOUSE}}{\text{AREA OF YARD}} = \frac{12 \times 10}{120 \times 310} = \frac{1}{310} \approx 0.003$$

9. (3 points) Write $5.\overline{36}$ as a ratio of two integers (i.e. a fraction) in lowest terms.

$F = 5.363636\ldots$

$100F = 536.363636\ldots$

$F = \quad 5.363636\ldots$

$99F = 531$

$F = \frac{531}{99} = \boxed{\frac{59}{11} = 5\frac{4}{11}}$

10. (2 points) Write 0.70053 in expanded form. Then write it as a fraction.

$7 \times 10^{-1} + 5 \times 10^{-4} + 3 \times 10^{-5}$

$$\frac{70053}{100000}$$

11. (3 points) Jasper picked up a bucket containing colored marbles and started selecting marbles at random with replacment. After making 25 selections, he had drawn 9 green marbles.

(a) Japser then stated that the probability of drawing a green marble is 9/25. Is this a theoretical or an experimental probability? Why?

THIS IS AN EXPERIMENTAL PROB. JASPER ASSIGNED THE PROBABILITY BY DOING THE EXPERIMENT:

$$\frac{\#\text{ OF GREEN DRAWS}}{\#\text{ OF DRAWS}} = \frac{9}{25}$$

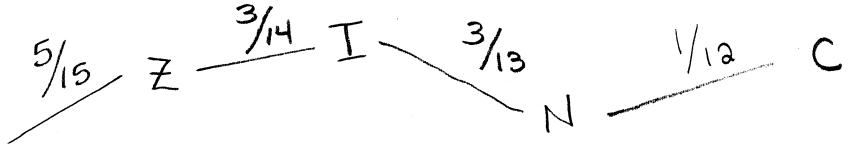
(b) Referring to your answer above, what would Jasper have to do to assign the other type of probability?

COUNT THE MARBLES... ASSUMING EACH MARBLE IS EQUALLY LIKELY, THEO. PROB. = $\frac{\#\text{ OF GREEN MARBS}}{\text{TOTAL \# OF MARBS}}$

(c) If the probability of drawing a blue marble is 15/25 and there are a total of 75 marbles in the bucket, how many are blue?

$$\frac{15}{25} = \frac{X}{75} \Rightarrow \boxed{X = 45}$$

12. (3 points) Four letters are selected one at a time, without replacement, from the word ZENZIZENZIZENIC. What is the probability of selecting the letters ZINC in that order?



PROB IS $\frac{5 \times 3 \times 3 \times 1}{15 \times 14 \times 13 \times 12} = \frac{45}{32760}$

$= \frac{1}{728}$

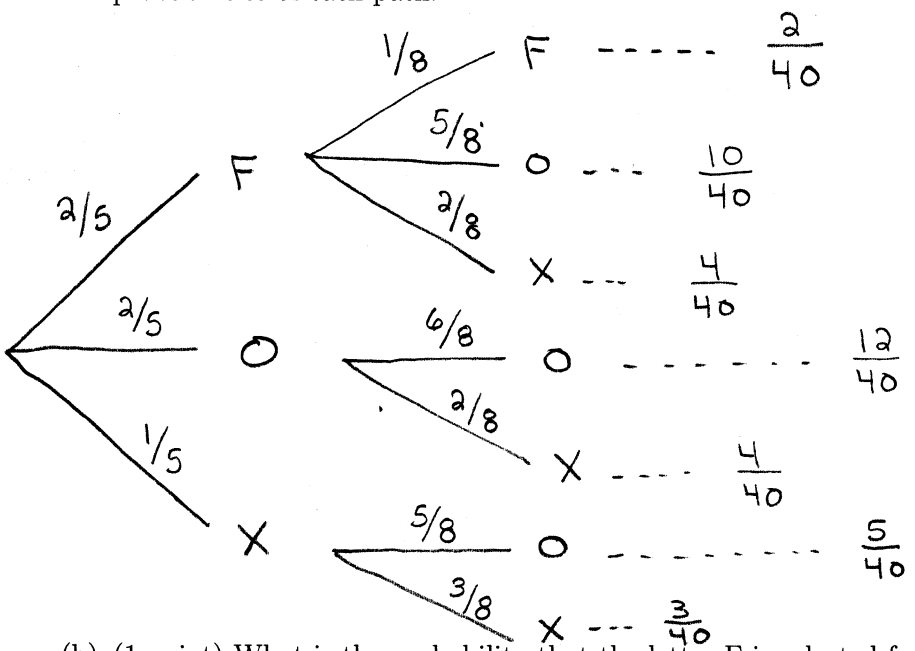
≈ 0.00137

13. A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.

F	F	O	O	X

O	O	O	O	O	X	X

(a) (3 points) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



(b) (1 point) What is the probability that the letter F is selected from box 2?

$$\frac{2}{40} = \frac{1}{20}$$

(c) (1 point) What is the probability of drawing the letter O or the letter X from box 2?

$$\frac{38}{40}$$

(d) (1 point) What is the sum of all the probabilities of the paths?

1 (HAS TO BE!)

14. (3 points) Indicate whether each statement is true or false.

(a) If A and B are mutually exclusive, then $P(A \cup B) = 0$.

FALSE, $P(A \cap B) = 0$

(b) If D is an impossible event, then $\overline{D}$ is a certain event.

TRUE

(c) If $P(X) = 0.4$ and $P(Y) = 0.3$, then it must be true that $P(X \cup Y) = 0.7$.

FALSE, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) \leq 0.7$

15. (4 points) Suppose A and B are events such that $P(A) = 0.46$, $P(B) = 0.68$, and $P(A \cup B) = 0.92$. Find each of the following.

(a) $P(A \cap B)$

$= P(A) + P(B) - P(A \cup B) = 0.46 + 0.68 - 0.92$

$= \boxed{0.22}$

(b) $P(\overline{A})$ $= 1 - 0.46 = \boxed{0.54}$

(c) $P(A \cup \overline{A})$ $= \boxed{1}$

(d) $P(\overline{A \cup B})$ $= 1 - 0.92 = \boxed{0.08}$

16. (3 points) Claudia and John own Jolly Maids Cleaning Service. Claudia can clean a client's house in 4 hr, while John can clean the same house in 5 hr. Working together, how long will it take them to clean the house?

CLAUDIA: $\frac{1/4 \text{ HOUSE}}{1 \text{ HR}}$

JOHN: $\frac{1/5 \text{ HOUSE}}{1 \text{ HR}}$

TOGETHER $\frac{(1/4 + 1/5) \text{ HOUSE}}{1 \text{ HR}} = \frac{9/20 \text{ HOUSE}}{1 \text{ HR}}$

$= \frac{1 \text{ HOUSE}}{\frac{20}{9} \text{ HR}}$

$\boxed{\frac{20}{9} \text{ HR} \approx 2.22 \text{ HR}}$