Math 206 - Test 1
February 7, 2018

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) A jar contains 6 blue marbles, 4 red marbles, and 2 green marbles. A marble is selected at random.

   (a) What is the sample space for this experiment?

   $S = \{b, r, g\}$

   (b) Is your sample space a uniform sample space? Explain.

   No. $P(\{b\}) = \frac{6}{12}$, $P(\{r\}) = \frac{4}{12}$, $P(\{g\}) = \frac{2}{12}$

   NOT EQUAL.

   (c) What is the probability that the marble is not red?

   $1 - \frac{4}{12} = \frac{8}{12}$, WHICH IS $\frac{6}{12} + \frac{2}{12}$

   (d) Is your probability above theoretical or experimental?

   (e) Instead of selecting one marble, suppose four marbles are selected (without replacement). What is the probability that at least two have the same color? Explain.

   100% = 1

   THERE ARE ONLY 3 COLORS. PICKING 4 MARBLES MUST GIVE AT LEAST TWO OF THE SAME COLOR.

2. (3 points) Each situation below describes a multistage experiment. Determine the best number of stages for each.

   (a) A pair of 12-sided dice are rolled.

   2 STAGES

   (b) A jar contains 8 quarters, 10 dimes, and 9 pennies. Three coins are selected at random without replacement.

   3 STAGES

   (c) Letters of the word *ENCYCLOPEDIA* are selected at random with replacement in an attempt to spell the name NANCY.

   5 STAGES

3. (3 points) The experiment is to select a letter at random from the word *MISSISSIPPI*.

   (a) Give an example of an event A with probability satisfying $0.5 < P(A) < 1.0$.

   $\{M,I,S,P\}$ $\frac{1}{11}, \frac{4}{11}, \frac{4}{11}, \frac{2}{11}$

   A = EVENT OF SELECTING A CONSONANT = $\{M,S,P\}$ $P(A) = \frac{7}{11} \approx 64\%$

   (b) Give an example of an impossible event.

   EVNT OF SELECTING $Q = \phi \Rightarrow$ PROB IS ZERO

   (c) Give an example of a certain event.

   $\{M,I,S,P\}$ SAMPLE SPACE HAS PROB 1.

4. (3 points) Four letters are selected one at a time, without replacement, from the word *ZENZIZENZIZENZIC*. What is the probability of spelling the word *NINE* (in order)?

   $\frac{3}{16}$ N $\frac{3}{15}$ I $\frac{2}{14}$ N $\frac{3}{13}$ E

   $\frac{3 \times 3 \times 2 \times 3}{16 \times 15 \times 14 \times 13} = \frac{54}{43680}$

5. (3 points) Indicate whether each statement is true (T) or false (F).

(a) F If A and B are mutually exclusive, then $P(A \cup B) = 0$.

→ $P(A \cap B) = 0$

(b) T If M is an impossible event, then $\overline{M}$ is a certain event.

(c) F If A and B are independent events, then $P(A|B) = P(B)$.

→ $P(A|B) = P(A)$

6. (5 points) A letter is selected at random from the word *watershed*. Let V be the event of selecting a vowel, C be the event of selecting a consonant, and F be the event of selecting a letter from the first half of the alphabet. Determine each probability.

$V = \{a, e\}$

$F = \{a, e, h, d\}$

$C = \{w, t, r, s, h, d\}$

(a) $P(V|F) = \frac{3}{5}$

↑ a, e, or e out of a, e, h, e, d

(b) $P(F|V) = \frac{3}{3}$

↑ a, e, h, e, or d out of a, e, e

(c) $P(F \cap V) = \frac{3}{9}$

$F \cap V = \{a, e\}$

↑ a, e, or e out of w, a, t, e, r, s, h, e, d

(d) $P(V \cup C) = 1$

$V \cup C = \{a, e, w, t, r, s, h, d\}$

↑ Vowel or consonant is all of them

(e) $P(\overline{F}) = \frac{4}{9}$

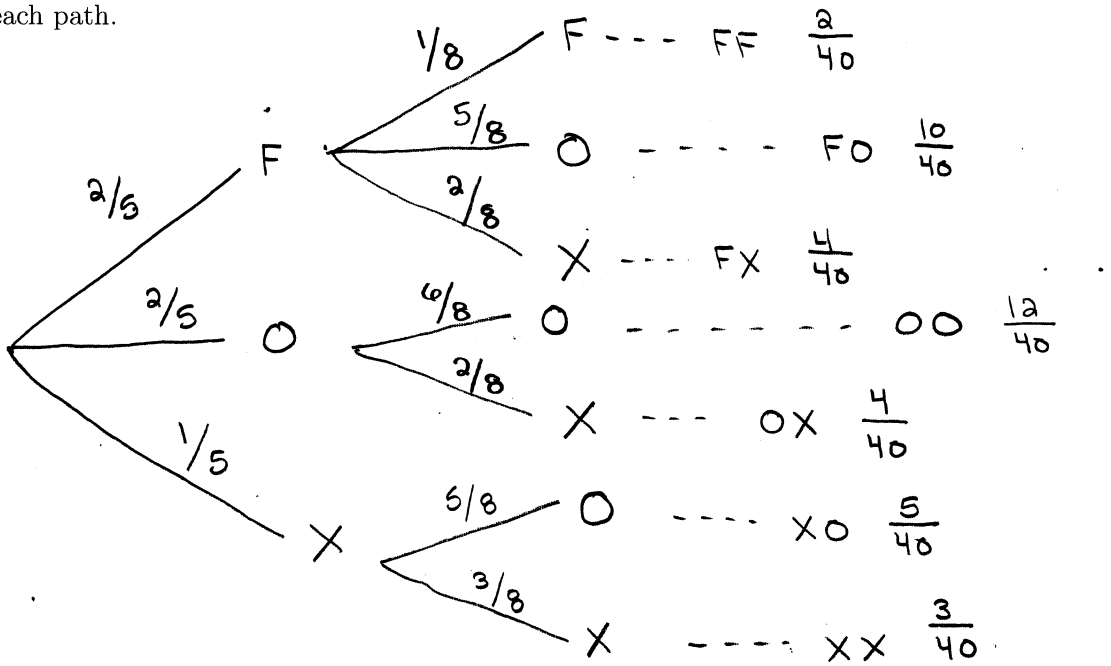
↑ w, t, r, s out of w, a, t, e, r, s, h, e, d

7. (10 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.

F	F	O	O	X

O	O	O	O	O	X	X

(a) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



(b) What is the probability of selecting at least one letter O?

FO, OO, OX, XO

$\frac{10}{40} + \frac{12}{40} + \frac{4}{40} + \frac{5}{40} = \boxed{\frac{31}{40}}$

(c) What is the probability of selecting at least one letter that is not F?

NOT FF

$1 - \frac{2}{40} = \boxed{\frac{38}{40}}$

(d) What is the sum of the probabilities of all the possible paths?

MUST BE $\boxed{1}$ OR $\frac{40}{40}$

8. (2 points) A PSC student is selected at random. Let A be the event that the student is taking a math class. Let B be the event that the student is a female. Are A and B disjoint (mutually exclusive)? Explain.

A & B ARE NOT EXCLUSIVE ...

THEY ARE FEMALE MATH STUDENTS.

9. (4 points) A card is selected at random from a standard deck. Let A be the event of drawing a red card. Let B be the event of drawing a jack. Determine $P(A)$ and $P(A|B)$. Are A and B independent? Explain.

$P(A) = \frac{26}{52} = \frac{1}{2}$

$P(A|B) =$ PROB OF DRAWING A RED CARD FROM THE 4 JACKS $= \frac{2}{4}$

$P(A) = P(A|B)$ so A & B ARE INDEPENDENT.

10. (4 points) Suppose A and B are events such that $P(A) = 0.46$, $P(B) = 0.68$, and $P(A \cup B) = 0.92$. Find each of the following.

(a) $P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.92 = 0.46 + 0.68 - \square$

$0.92 = 1.14 - \square$ → 0.22

(b) $P(\overline{A})$

$1 - P(A) =$ 0.54

(c) $P(A \cup \overline{A}) =$ 1

(d) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.22}{0.68} \approx 0.32$

11. (3 points) The sample space for an experiment is $\{1, 2, 3, 4, 5, 6\}$. Is it necessarily true that the probability of obtaining an even number is 3/6? Explain.

↳ No, THE OUTCOMES IN THE SAMPLE SPACE ARE NOT NECESSARILY EQUALLY LIKELY.

12. (3 points) The local Chuck E. Cheese restaurant has a ball pit filled with colored balls. A child selects a single ball at random. The probability of selecting a green ball is 21/80. Is it possible that the probability of selecting a red ball is 3/4? Explain.

No! IF SO, THE PROB OF GREEN OR RED WOULD BE $\frac{21}{80} + \frac{3}{4} = \frac{81}{80} > 1$.

13. (2 points) Suppose the wicked witch of the east is hanging out at a random location in her big yard which measures 120 ft by 310 ft. Dorothy Gale's little house measures 12 ft by 10 ft, and a tornado is about to hurl the house into the witch's yard. What is the probability that the witch is smashed by the falling house?

$$\frac{\text{AREA OF HOUSE}}{\text{AREA OF YARD}} = \frac{12 \times 10}{120 \times 310} = \frac{120}{37200} \approx 0.0032$$