Math 216 - Quiz 6

November 3, 2010

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Solve the following initial value problem:

$$x'' - 6x' + 34x = 0; \quad x(0) = 3,\ x'(0) = 5$$

$r^2 - 6r + 34 = 0$

$r = \dfrac{6 \pm \sqrt{36 - 4(1)(34)}}{2}$

$r = \dfrac{6 \pm 10i}{2} = 3 \pm 5i$

$\alpha = 3,\ \beta = 5$

$x(t) = c_1 e^{3t} \cos 5t + c_2 e^{3t} \sin 5t$

$x(0) = 3 \Rightarrow 3 = c_1$

$x'(t) = 3c_1 e^{3t} \cos 5t - 5c_1 e^{3t} \sin 5t + 3c_2 e^{3t} \sin 5t + 5c_2 e^{3t} \cos 5t$

$x'(0) = 5 \Rightarrow 5 = 3c_1 + 5c_2 = 9 + 5c_2$

$\Rightarrow c_2 = -\frac{4}{5}$

$$\boxed{x(t) = 3e^{3t} \cos 5t - \frac{4}{5} e^{3t} \sin 5t}$$

2. (4 points) Solve the following differential equation:

$$y'' - 2y' - 3y = 24e^{3x}$$

① Homo eq:

$y'' - 2y' - 3y = 0$

$r^2 - 2r - 3 = 0$

$(r-3)(r+1) = 0$

$r = 3,\ r = -1$

$y_h(x) = c_1 e^{3x} + c_2 e^{-x}$

② Non Homo:

$y'' - 2y' - 3y = 24e^{3x}$

$g(x) = 24e^{3x}$

$y_p(x) = x^s (Ae^{3x})$

Choose $s = 1$.

$y_p(x) = Axe^{3x}$

$y_p'(x) = Ae^{3x} + 3Axe^{3x}$

$y_p''(x) = 3Ae^{3x} + 3Ae^{3x} + 9Axe^{3x}$

$6Ae^{3x} + \cancel{9Axe^{3x}} - 2Ae^{3x} - \cancel{6Axe^{3x}} - \cancel{3Axe^{3x}} = 24e^{3x}$

$4Ae^{3x} = 24e^{3x} \Rightarrow A = 6$

$$\boxed{y(x) = c_1 e^{3x} + c_2 e^{-x} + 6xe^{3x}}$$

3. (3 points) Solve: $y'' - 6y' + 8y = 6x^2 + 3$

① Homo eq: $y'' - 6y' + 8y = 0$

$r^2 - 6r + 8 = 0$

$(r-2)(r-4) = 0$

$r = 2, \; r = 4$

$y_h(x) = c_1 e^{2x} + c_2 e^{4x}$

② Non Homo eq:

$y'' - 6y' + 8y = 6x^2 + 3$

$g(x) = 6x^2 + 3$

$y_p(x) = x^s (Ax^2 + Bx + C)$

$s = 0$

$y_p(x) = Ax^2 + Bx + C$

$y_p'(x) = 2Ax + B$

$y_p''(x) = 2A$

$2A - 6(2Ax + B) + 8(Ax^2 + Bx + C)$

$= 6x^2 + 3$

$8A = 6 \implies A = \frac{3}{4}$

$-12A + 8B = 0 \implies -9 + 8B = 0 \Rightarrow B = \frac{9}{8}$

$2A - 6B + 8C = 3 \implies \frac{6}{4} - \frac{54}{8} + 8C = 3 \Rightarrow 8C = \frac{24}{8} - \frac{12}{8} + \frac{54}{8}$

$C = \frac{66}{64}$

$y_p(x) = \frac{3}{4}x^2 + \frac{9}{8}x + \frac{33}{32}$

$y(x) = c_1 e^{2x} + c_2 e^{4x} + \frac{3}{4}x^2 + \frac{9}{8}x + \frac{33}{32}$